Elliptic flow from colour strings

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I. Strings propagating in the transverse plane

Strings can propagate in the transverse space and so be characterized by a vector in this space rather than a point. In the Regge language this corresponds to assuming that the pomeron slope $\alpha'$ is substantially different from zero, in contrast to the original assumption that the slope is quite small. In the 3-dimensional space the string becomes not orthogonal to the transverse plane and one may expect an anisotropic emission of particles in this plane and a non-zero elliptic flow. This mechanism is quite similar to the one considered by K.G.Boreskov, A.B.Kaidalov and O.V.Kancheli [Eur.Phys. J. C 58 (2008) 445] with the anisotropy created already in the initial emission.
I.1. The model

The probability to form a string stretched between points $\beta_1$ in the projectile nucleus and $\beta_2$ in the target one is given by the distribution in $\beta = \beta_1 - \beta_2$ of the soft pomeron

$$P(y, \beta) = \frac{1}{4\pi \alpha' y} e^{\epsilon y - \beta^2/(4\alpha'y)}.$$  \hfill (1)

The string has also some dimension along the $z$-axis. We assume that the string is located in the $xz$-plane and its direction defines the $z'$ axis in the primed coordinate system $(x', y', z')$. The $y'$ axis is assumed to coincide with the $y$ axis in the original system and the $x'$ axis is taken to be orthogonal to $y'$ and $z'$ axes. (see Fig. 1).
The string in the 3-dimensional space. The shown angle is $\theta = \pi/2 - \theta'$.
One may assume that particles are only emitted isotropically in the plane orthogonal to the string direction (the Schwinger mechanism). So in the primed system the probability to emit a particle with momentum \( p' \) is

\[
\mu'(y, p') = \delta(p'_z) \frac{\xi}{\pi} e^{-\xi(p'^2_x + p'^2_y)}.
\]  

(2)

In the original system the distribution of emitted particles in \( p_\perp \) is

\[
\mu(y, p_\perp) = \frac{\xi}{\pi \cos \theta'} e^{-\xi(p'^2_y + p'^2_x/\cos^2 \theta')}.
\]  

(3)

with

\[
\theta' = \arctan \frac{\beta}{\Delta z}.
\]  

(4)

On the average \( \beta \sim \sqrt{\alpha'y} \) and \( \Delta z \sim e^{-y/2} \). So if \( \theta' = \frac{\pi}{2} - \theta \) and \( \alpha' \neq 0 \) then \( \theta \) is small and diminishes with energy:

\[
\theta \sim \frac{e^{-y/2}}{\sqrt{\alpha'y}}.
\]  

(5)
With $\alpha' \neq 0$ emission is anisotropic and one expects a non-zero elliptic flow effect.

A more general assumption is that emission along the string axis is not zero but simply different from emission in the transverse plane,

$$\mu(y, p') = \sqrt{\eta} \left( \frac{\xi}{\pi} \right)^{3/2} e^{-\xi(\eta p'_x^2 + p'_y^2 + p'_z^2)}, \quad (6)$$

If $\eta > 1$ the emission along the string is damped.

The final distribution in $p_\perp$ is obtained after integration over $p_z$

$$\mu(y, p_\perp) = \frac{\xi \sqrt{\gamma}}{\pi} \exp \left( -\xi(p_y^2 + \gamma p_x^2) \right), \quad (7)$$

where the final eccentricity parameter $\gamma$ is

$$\gamma = \frac{\xi}{\cos^2 \theta + \xi \sin^2 \theta} \quad (8)$$
The final distribution of emitted particles at fixed impact parameter $b$ is

$$M(b, p) = \int d^2 \beta_1 d^2 \beta_2 P_A(\beta_1) P_B(\beta_2)$$

$$(\beta_1 - \beta_2) \mu(y, \beta, p_\perp),$$

(9)

where $\mu$ is given by (7) with the $x$-axis along the direction of the string, that is, along $\beta = \beta_1 - \beta_2$. $P_A$ is the probability to find a string attached to point $\beta_1$ in the projectile nucleus A

$$P_A(\beta_1) = \int d^2 b_1 T_A(b_1) t(\beta_1 - b_1),$$

(10)

where $T_A(b)$ is the nuclear profile function and $t(b)$ is the distribution of strings in the nucleon, $p(\beta_1 - \beta_2)$ is the probability to find a string stretched between the points $\beta_1$ and $\beta_2$

$$p(\beta) = \frac{1}{4\pi \alpha' y} \exp \left( - \frac{\beta^2}{4\alpha' y} \right).$$

(11)
I.2. Strings homogeneously distributed

For simplicity we assume

\[ P_A(\beta_1) = N_s \frac{\theta(R_A - \beta_1)}{\pi R_A^2}, \]

(12)

where \( N_s \) is the number of strings and \( R_A \) is the nucleus radius. Then for collisions of two identical nuclei, up to unimportant coefficient,

\[ M(b, p, \phi) = \int_0^{\beta_m} d\beta \int_{-\pi}^{\pi} d\chi \left( 2\zeta - \sin(2\zeta) \right) e^{-\beta^2/\beta_0^2} e^{-\xi p^2(\sin^2(\chi+\phi)+\gamma \cos^2(\chi+\phi))}, \]

(13)

where

\[ \beta_m = 2R_A + b, \quad \beta_0 = 4\alpha' y, \]

\[ \zeta = \arccos \frac{\sqrt{b^2 + \beta^2 - 2b\beta \cos \chi}}{2R_A} \]

and \( \xi \) is the inverse string tension which determines the distribution in momenta.
We take $\alpha' = 0.2 \text{ GeV}^{-2}$ and $\xi = 0.25 \text{ GeV}^{-2}$. The only left parameter is excentricity $\gamma$.

One finds that if $\gamma > 1$ then $v_2 < 0$ and if $\gamma < 1$ then $v_2 > 0$. So if the elliptic flow comes only from string propagation in the transverse plane, the experimental data exclude $\gamma > 1$ and thus the naive picture in which the string only emits particles in the plane transverse to its direction, as in the Schwinger picture. Rather the opposite: the string emits particles predominantly along its direction, which leads to $\gamma < 1$.

However both with $\gamma < 1$ and $\gamma > 1$ the magnitude of the elliptic flow turns out to be quite small at accessible rapidities. Our results for Au-Au collisions at $y = 10$ and with $\gamma = 0.1$ are shown in Figs. 2 (the $b$-dependence of $v_2$) and 3. (its $p$-dependence)
The qualitative behaviour of both $b$- and $p$-dependence agrees with the experimental findings. However values of $v_2$ at not too peripheral collisions are ten to twenty times lower than the data. Only in the limiting case of a very small overlap these values become comparable to the observed ones.

In Fig. 4 we show the elliptic flow for $\gamma = 10$, when the resulting $v_2$ is negative. Its values turn out to be practically independent of $p$. The magnitude of $|v_2|$ and its behaviour with $b$ are very similar to what we have obtained for values of $\gamma$ smaller than unity. Again $|v_2|$ reaches value of the order of several percents only at very peripheral collisions.
Elliptic flow coefficient $v_2$ with $\gamma = 0.1$ as a function of $b$ for Au-Au collisions at $y = 10$. The curves from bottom to top correspond to $p = 0.5, 1.1.5$ and 2 GeV.
Elliptic flow coefficient $v_2$ with $\gamma = 0.1$ as a function of $p$ at $b = 6$ fm for Au-Au collisions at $y = 10$.

Elliptic flow coefficient $|v_2|$ with $\gamma = 10$ as a function of $b$ for Au-Au collisions at $y = 10$. 
II. String fusion as a source of elliptic flow

II.1 The model

String fusion \(\rightarrow\) percolation at the string density in the transverse space \(\rho \sim 1.1 - 1.2\).

Two ingredients to have a non-zero elliptic flow.

- String fusion has to generate clusters which are azimuthally asymmetric and emit particles anisotropically.

- The distribution of these clusters in the transverse plane has to be also azimuthally asymmetric. This latter phenomenon can only occur if the clusters are large enough to feel the asymmetric form of the overlapping region in the collision. Such clusters arise in the process of percolation of fused strings.
Fused strings are naturally asymmetric azimuthally. As a source of asymmetry in the emission of particles in different azimuthal directions, one may consider quenching of emitted particles as they pass through the fused string area. If the emitted particle propagates inside the string along a path of length $l$ the probability to see the particle of momentum $p$ outside the string is proportional to

$$P(p, l) = e^{-\frac{\xi p^2}{1-\sigma l}}, \quad (14)$$

where $\sigma$ characterizes the loss of the transverse energy per unit length. If the fused string is not symmetric, then the length $l$ will depend on the direction of emission.

If additionally the nuclear overlap is not symmetric in the azimuthal angle, then fused strings of different forms will be created with different probabilities. This will give rise to a non-zero elliptic flow. A clear limiting case is when $\rho \gg 1$ and all strings fuse into one which occupies all the overlap area and so is unsymmetric if the latter is.
II.2. Simplified approach.

In the limit $\rho \to \infty$ the fused string occupies all the overlap area and its form coincides with the latter. For collision of identical nuclei it is a symmetrical almond. In our simplified approach we assume that at finite (but large $\rho$) fused strings are distributed homogeneously in the overlap area and preserve the almond form. The emission probability from a string will depend on the initial emission point $(x_0, y_0)$ and angle $\chi$ between the direction of the emission and minor axis of the string.
The total emission probability at given $p$ and $\chi$, up to a constant factor, is

$$P(p, \chi) = \int dx_0 dy_0 e^{-\frac{\xi p^2}{1-\sigma l(\chi, x_0, y_0)}}. \quad (15)$$

The total emission probability at given $\chi$, up to a constant factor,

$$P(\chi) = \int dx_0 dy_0 \left( 1 - \sigma l(\chi, x_0, y_0) \right). \quad (16)$$

The fused string can be generally directed at any angle $\theta$ respective to the direction of $b$ with the probability $D(\theta)$ So the final distribution in the transverse momentum of particles emitted from this fused string will be obtained as

$$d\sigma \over dpdpd\phi = C \int d\theta D(\theta) P(p, \phi - \theta) \quad (17)$$

and the distribution in $\phi$ as:

$$d\sigma \over d\phi = C_1 \int d\theta D(\theta) P(\phi - \theta). \quad (18)$$

Obviously if the distribution of strings in $\theta$ is isotropic, then integration over $\theta$ will eliminate any dependence on $\phi$. 

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At $\rho \to \infty$

$$D(\theta) = \delta(\theta)$$

(19)

and then

$$\frac{d\sigma}{pdpd\phi} = CP(p, \phi),$$

(20)

$$\frac{d\sigma}{d\phi} = C_1 P(\phi)$$

(21)

where the almond of the string occupies the whole overlap area

At finite $\rho$ strings occupy only part of the overlap area given by factor

$$F(\rho) = 1 - e^{-\rho}.$$  

(22)

so, on the average, an emitted particle has to pass through the string matter length $l$ which is smaller than the corresponding length at $\rho >> 1$ by factor

$$\kappa_1(\rho) = \sqrt{F(\rho)}.$$  

(23)
The average string tension will be greater than for the single string by factor $\kappa(\rho)^{-1}$ where

$$\kappa(\rho) = \sqrt{\frac{1 - e^{-\rho}}{\rho}} = \frac{\kappa_1}{\sqrt{\rho}}. \quad (24)$$

The distribution in the transverse momentum will then be given by the same formulas (20) and (21) with the appropriate rescaling of $l$ and $\xi$

$$\frac{d\sigma}{pdpd\phi} = CP\left(p, \phi, \xi \to \xi \kappa, l \to \kappa_1 l\right) \quad (25)$$

$$\frac{d\sigma}{d\phi} = C_1 P\left(\phi, a \to a \kappa, l \to \kappa_1 l\right) \quad (26)$$

Our model contains a single new parameter $\sigma$ which characterizes the loss of energy in passing through the string field.

$$\sigma R_A = \sigma_0 A^{1/3} \quad (27)$$

where $\sigma_0$ is a dimensionless and $A$ independent parameter to be extracted from the experimental data, which give $\sigma_0 = 0.09$
Values of $\rho$ were taken from T.J.Tarnowsky et al. [arXiv:nucl-ex/0606019] for Au-Au collisions at 62.4 and 200 GeV.

Percolation parameter as a function of impact parameter for Au-Au collisions at 62.4 (lower curve) and 200 GeV.

For 5500 GeV, based on our earlier estimates, we assumed the values of $\rho(b)$ to be twice their values at 200 GeV.
$v_2$ as a function of $b$ at 62.4 (lower curve), 200 and 5500 GeV (upper curve).

$v_2$ as a function of $p$ at 200 GeV for peripheral (uppermost), mid-central and central (lowest) collisions.
$v_2$ as a function of $p$ 5500 GeV for peripheral (uppermost), mid-central and central (lowest) collisions
II.3. Monte-Carlo simulations

To fully take into account different forms and distribution of fused trings we performed Monte-carlo simulations of string fusion.

One models strings by discs of a given radius $r_0$. To simplify we consider the case of a constant profile function of each nucleus. Then in an event $N$ discs are assumed to be homogeneously distributed in the nuclear overlap area. The number of discs is to be chosen in agreement with the observed value of the percolation parameter $\rho$ given by

$$\rho = \frac{N\Omega_0}{\Omega} \quad (28)$$

where $\Omega_0 = \pi r_0^2$ is the transverse area of a string and $\Omega$ is the overlap area. $R = A^{1/3}R_0$ with $R_0 = 1.2$ fm and $r_0 = 0.3$ fm, for $Au = Au$ collisions at 62.4 and 200 GeV one gets the number of strings given in Table 1 for different excentricities $\zeta = \arccos(b/2R_A)$. 

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Table 1.

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<th>$\zeta$</th>
<th>$\rho_{62}$</th>
<th>$\rho_{200}$</th>
<th>$N_{62}$</th>
<th>$N_{200}$</th>
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</table>
The strings modelled by discs may overlap and form clusters of different number $n$ of fused strings and form. Observed particles are emitted from each cluster with the average multiplicity and momentum squared

$$\mu_{nk} = \sqrt{\frac{n\Omega_{nk}}{\Omega_0}}\mu_0, \quad p^2_{nk} = \sqrt{\frac{n\Omega_0}{\Omega_{nk}}}p^2_0$$

(29)

for the $k$-th cluster of $n$ fused strings. Here $\Omega_{nk}$ is the transverse area of the cluster and $\mu_0$ and $p^2_0$ refer to the simple string.

Each particle emitted from a given point in a cluster has to pass a certain path in the overlap area before being observed. A part of it has to pass through the same or different clusters and so loses its energy.
The average length $l_{nk}$ travelled by the particle emitted from the $k$-th cluster of $n$ strings depends both on the distribution of clusters and on the direction of the emission. Due to the azimuthal asymmetry of the cluster distribution following from the asymmetry of the overlap area the average distribution of emitted particles will depend on the azimuthal angle and lead to a non-vanishing elliptic flow.
The final distribution of emitted particles in the transverse momentum and azimuthal angle will be given as a sum

\[ P(p, \phi) = C \sum_{n,k} \mu_{nk} e^{\frac{-\xi p^2}{1-\sigma_{nk}}} \]  (30)

and the distribution in the azimuthal angle only

\[ P(\phi) = C_1 \sum_{n,k} \mu_{nk} (1 - \sigma l_{nk}) \]  (31)

We again adjusted the quenching coefficient \( \sigma_0 \) to fit the experimental value of \( v_2 \) integrated over the transverse momenta for Au-Au collisions at 200 GeV in mid-central events. The adjusted coefficient turned out to be twice larger than for our simplified treatment in the previous sections: \( \sigma_0 = 0.2 \).
Monte-Carlo $v_2(b)$ at 62.4 (lower curve), 200 and 5500 GeV (upper curve)

Monte-Carlo $v_2(p)$ at 200 GeV for mid-central (uppermost), peripheral and central (lowest) collisions.
Monte-Carlo $v_2(p)$ at 5500 GeV for mid-central (uppermost), peripheral and central (lowest) collisions.
III. Conclusions

- The colour string model with fusion and percolation can successfully describe the observed elliptic flow in high-energy heavy-ion collisions due to anisotropy of the string emission spectra in the azimuthal direction.

- This anisotropy coming from string propagation in the transverse plane due to a non-zero pomeron slope plays a minor role at accessible energies because the distance travelled by the string in the transverse plane turns out to be small.

- Quenching of the emitted partons in the strong colour field inside the string gives rise to anisotropy, which, upon adjusting the parameter of quenching, allows to describe the data quite well both in their centrality dependence and their transverse momentum dependence.
Our calculations show that the dependence of the elliptic flow on $p_T$ at LHC is almost equal to that at RHIC, in agreement with the recent ALICE data. This agreement gives support to the percolation approach as a microscopic picture for the hydrodynamical description.

The reason for this weak energy dependence is that the distribution in $\phi$ depends only on parameter $\kappa_1$, which quickly goes to unity as soon as the percolation parameter grows above the percolation threshold. So in our picture values of $v_2$ saturate with the growth of energy as soon as the percolation parameter becomes large.