\( \pi \text{DA and } \pi \text{FFs in QCD Sum Rules} \)

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Outline:

- Pion Distribution Amplitude (DA) in QCD
- QCD SRs with Nonlocal Condensates for Pion DA
- Light-Cone Sum Rules (LCSRs) Approach
- LCSRs Results for $\pi\gamma$ Transition Form Factor (FF)
- Fitting Pion DA — Confidential Regions
- Confidential Region for Pion DA Moments vs. Lattice QCD
- Fit Results and Pion DA Models
- Drell–Yan $\pi N$-Process and Pion DA
- Conclusions
# Publications

**Authors:** A. B., S. Mikhailov, A. Pimikov, N. Stefanis, O. Teryaev

### Publications:

- A. B., S. M., N. S.  
  PLB 508 (2001) 279
- A. B., S. M., N. S.  
  PRD 67 (2003) 074012
- A. B., S. M., N. S.  
  PLB 578 (2004) 91
- A. B., S. M., N. S.  
  PRD 73 (2006) 056002
- A. B., A. P.  
  APPB 37 (2006) 3627
- A. B., O. T., N. S.  
  PRD 76 (2007) 074032
- A. B., A. P., N. S.  
  PRD 79 (2009) 093010
- S. M., N. S.  
  NPB 821 (2009) 291
- S. M., N. S.  
  MPLA 24 (2009) 2858
- A. B., S. M., N. S.  
  APPB PS3 (2010) 943
- A. B., S. M., A. P., N. S.  
  PRD 84 (2011) 034014
- A. B., S. M., A. P., N. S.  
- N. S.  
  arXiv:1109.2718 [hep-ph]
Pion DA
(Distribution Amplitude)
in QCD
Pion distribution amplitude (DA)

- Matrix element of nonlocal axial current on light cone

\[
\langle 0 | \bar{d}(z) \gamma_\mu \gamma_5 E(z, 0) u(0) | \pi(P) \rangle \bigg|_{z^2 = 0} = i f_\pi P_\mu \int_0^1 dx \ e^{ix(zP)} \varphi^{\text{Tw-2}}_\pi(x, \mu^2)
\]

- Gauge-invariance due to Fock–Schwinger string:

\[
E(z, 0) = \mathcal{P} e^{ig \int_0^z A_\mu(\tau) d\tau^\mu}
\]

- Physical meaning of $\varphi_\pi(x; \mu^2)$ — amplitude for transition $\pi \rightarrow u + d$
It is convenient to represent the pion DA:

\[ \varphi_\pi(x; \mu^2) = \varphi^{As}(x) \times \]
\[ \times \left[ 1 + a_2(\mu^2) C_2^{3/2}(2x - 1) + a_4(\mu^2) C_4^{3/2}(2x - 1) + \ldots \right] \]

where \( C_n^{3/2}(2x - 1) \) are the Gegenbauer polynomials (1-loop eigenfunctions of ER-BL kernel)
It is convenient to represent the pion DA:

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where $C_n^{3/2}(2x - 1)$ are the Gegenbauer polynomials (1-loop eigenfunctions of ER-BL kernel)

That means

$$\{a_2(\mu^2), a_4(\mu^2), \ldots\} \Leftrightarrow \varphi_\pi(x; \mu^2)$$
**Representation of Pion DA**

- It is convenient to represent the pion DA:
  \[
  \varphi_\pi(x; \mu^2) = \varphi_{As}(x) \times \times \left[ 1 + a_2(\mu^2)C_2^{3/2}(2x - 1) + a_4(\mu^2)C_4^{3/2}(2x - 1) + \ldots \right]
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  where \( C_n^{3/2}(2x - 1) \) are the Gegenbauer polynomials (1-loop eigenfunctions of ER-BL kernel)

- That means
  \[
  \{ a_2(\mu^2), a_4(\mu^2), \ldots \} \Leftrightarrow \varphi_\pi(x; \mu^2)
  \]

- ER-BL solution at 2-loop level
  - Mikhailov & Radyushkin; 1986
  - Müller; 1994–95
  - A.B. & Stefanis; 2005
NLC QCD SRs
for
Pion DA
Illustration of NLC-model:
\[ \langle \bar{q}(0)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle e^{-|z|^2 \lambda_q^2 / 8} \]
Illustration of NLC-model: \[ \langle \bar{q}(0)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle e^{-|z|^2 \lambda_q^2/8} \]

A single scale parameter \( \lambda_q = \langle k^2 \rangle \) characterizing the average momentum of quarks in QCD vacuum:

\[
\lambda_q^2 = \begin{cases} 
0.4 \pm 0.1 \text{ GeV}^2 & \text{[QCD SRs, 1987]} \\
0.5 \pm 0.05 \text{ GeV}^2 & \text{[QCD SRs, 1991]} \\
0.4 - 0.5 \text{ GeV}^2 & \text{[Lattice, 1998-2002]}
\end{cases}
\]
Non-Local Condensates in QCD SR

Illustration of NLC-model: $\langle \bar{q}(0)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle e^{-|z^2|\lambda_q^2/8}$

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Correlation length $\lambda_q^{-1} \sim \rho$-meson size
Non-Local Condensates in QCD SR

Illustration of NLC-model: $\langle \bar{q}(0)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle e^{-|z|^2/\lambda_q^2}$

A single scale parameter $\lambda_q^2 = \langle k^2 \rangle$ characterizing the average momentum of quarks in QCD vacuum:

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0.4 \pm 0.1 \text{ GeV}^2 & \text{[ QCD SRs, 1987 ]} \\
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0.4 - 0.5 \text{ GeV}^2 & \text{[ Lattice, 1998-2002 ]}
\end{cases}
\]

Correlation length $\lambda_q^{-1} \sim \rho$-meson size

Possible to include second ($\Lambda \simeq 450$ MeV) scale with

\[
\langle \bar{q}(0)q(z) \rangle \bigg|_{|z| \gg 1 \text{ Fm}} \sim \langle \bar{q}q \rangle e^{-|z|\Lambda} \quad \text{(not included here)}
\]
Axial-axial correlator

We study correlator:

$$\Pi_{\mu\nu}^N = i \int d^4x \ e^{iqx} \langle 0 | T \left[ J_{\mu 5}^N(0) J_{\nu 5}^+(x) \right] | 0 \rangle$$

of two axial currents

$$J_{\mu 5}^N(0) = \bar{d}(0) \gamma_\mu \gamma_5 [ -in\nabla]^N u(0) \ ; \ J_{\nu 5}^+(x) = \bar{u}(x) \gamma_\nu \gamma_5 d(x)$$

corresponding to charged $\pi$-meson. Current $J_{\mu 5}^N(0)$ produces

$$\langle 0 \ | \ J_{\mu 5}^N(0) \ | \ \pi(P) \rangle = i f_\pi P_\mu (nP)^N \int_0^1 dx \ x^N \varphi_\pi(x)$$
Axial-axial correlator

We study correlator:

$$\Pi_{\mu\nu}^N = i \int d^4x \, e^{i q x} \langle 0 \left| T \left[ J_{\mu 5}^N(0) J_{\nu 5}^+(x) \right] \right| 0 \rangle$$

of two axial currents

$$J_{\mu 5}^N(0) = \bar{d}(0) \gamma_{\mu} \gamma_5 \left[ -in \nabla \right]^N u(0) ; \quad J_{\nu 5}^+(x) = \bar{u}(x) \gamma_{\nu} \gamma_5 d(x)$$

corresponding to charged $\pi$-meson. Current $J_{\mu 5}^N(0)$ produces

$$\langle 0 \mid J_{\mu 5}^N(0) \mid \pi(P) \rangle = i f_\pi P_\mu (nP)^N \langle x^N \rangle_\pi$$
Here is example of QCD SR with Non-Local Condensates

\[
f_{\pi}^{2} \phi_{\pi}(x) = \int_{0}^{s_{0}} \rho_{\text{pert}}(x; s) e^{-s/M^2} ds + \frac{\alpha_{s} \langle GG \rangle}{24\pi M^2} \phi_{G}(x; \Delta) \\
+ \frac{16\pi\alpha_{s} \langle \bar{q}q \rangle^2}{81M^4} \sum_{i=2V,3L,4Q} \phi_{i}(x; \Delta)
\]

Local limit: \( \lambda_{q}^{2}/M^2 \equiv \Delta \rightarrow 0 \),

\[
\phi_{G}(x; \Delta = 0) = [\delta(x) + \delta(1 - x)] \\
\phi_{2V}(x; \Delta = 0) = [x\delta'(1 - x) + (1 - x)\delta'(x)] \\
\phi_{4Q}(x; \Delta = 0) = 9[\delta(x) + \delta(1 - x)]
\]
NLC contributions to QCD SR

Examples for Gaussian NLC with a single parameter $\lambda_q^2$

Local limit: $\lambda_q^2/M^2 \equiv \Delta \rightarrow 0$,

$$\varphi_{4Q}^{\text{loc}}(x) \equiv \lim_{\Delta \rightarrow 0} \varphi_{4Q}^{\text{NLC}}(x; \Delta) = 9[\delta(x) + \delta(1 - x)]$$
NLC SRs for pion DA

Moments $\langle \xi^N \rangle_\pi = \int_0^1 \varphi_\pi(x) (2x - 1)^N dx$ at $\mu^2 \approx 1$ GeV$^2$

These $\langle \xi^N \rangle_\pi$ values allow one to restore DA $\varphi_\pi(x)$

from NLC SRs

$\lambda_q^2 = 0.4$ GeV$^2$:

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produce bunch of self-consistent 2-parameter models $\varphi_\pi(x)$ at $\mu^2 \simeq 1 \text{ GeV}^2$:

$$\varphi_\pi(x) = \varphi^{As}(x) \left[ 1 + a_2 C_2^{3/2} (2x - 1) + a_4 C_4^{3/2} (2x - 1) \right]$$

\[ \chi^2 \approx 0.001 \]
\[ \langle x^{-1} \rangle^{SR} = 3.30(30) \]
BMS [PLB (2001)]: at $\mu^2 \simeq 1$ GeV$^2$

- $\lambda_q^2 = 0.4$ GeV$^2$,
- $\left\langle x^{-1} \right\rangle_{\pi}^{SR} = 3.3 \pm 0.3$,
- $\left\langle x^{-1} \right\rangle_{\pi}^{b.f.} = 3.17$

The moment $\left\langle x^{-1} \right\rangle_{\pi}^{SR}$ could be determined only in NLC SRs because end-point singularities absent.
$\varphi_\pi(x)$

BMS DA is end-point suppressed!
$\varphi_\pi(x)$

Curves | DAs
--- | ---
--- | CZ
--- | Asymp.

CZ DA: end-point enhancement
BMS vs CZ distribution amplitude

\[ \varphi_\pi(x) \]

Curves DAs
- CZ
- BMS
- Asymp.

BMS bunch is 2-humped, but end-point suppressed!
Histograms for inverse moment $\langle x^{-1} \rangle_\pi$

Contributions of different DAs to inverse moment $\langle x^{-1} \rangle_\pi$, calculated as $\int_{x}^{x+0.02} \phi(x) dx$ and normalized to 100%, for:

(a) CZ and BMS DAs;  
(b) Asympt. and BMS DAs.

In BMS case region $x \leq 0.1$ contributes even less than in Asymptotic DA case.
Estimated bunches of pion DAs for different values of $\lambda_q^2$. 

- $\lambda_q^2 = 0.6 \text{ GeV}^2$
- $\lambda_q^2 = 0.5 \text{ GeV}^2$
- $\lambda_q^2 = 0.4 \text{ GeV}^2$

$\mu^2 = 1.35 \text{ GeV}^2$
NLO Light-Cone SRs $\Rightarrow$
CLEO data on $F_{\gamma\gamma^*\pi}(Q^2) \Rightarrow$
Constraints on Pion DA
\( \gamma^* \gamma \rightarrow \pi : \text{Why Light-Cone Sum Rules?} \)

For \( Q^2 \gg m_{\rho}^2, \quad q^2 \ll m_{\rho}^2 \) pQCD factorization valid only in leading twist and higher twists are of importance [Radyushkin–Ruskov, NPB (1996)].

Reason: if \( q^2 \rightarrow 0 \) one needs to take into account interaction of real photon at long distances \( \sim O(1/\sqrt{q^2}) \)

pQCD is OK

LCSRs should be applied
\[ \gamma^* \gamma \rightarrow \pi : \text{Why Light-Cone Sum Rules?} \]

For \( Q^2 \gg m^2_\rho \), \( q^2 \ll m^2_\rho \) pQCD factorization valid only in leading twist and higher twists are of importance\([\text{Radyushkin–Ruskov, NPB (1996)}]\).

Reason: if \( q^2 \rightarrow 0 \) one needs to take into account interaction of real photon at long distances \( \sim O(1/\sqrt{q^2}) \)

To account for long-distance effects in pQCD one needs to introduce light-cone DA of real photon
Khodjamirian [EJPC (1999)]: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in $q^2$

$$F_{\gamma^*\gamma^*\pi}(Q^2, q^2) = \frac{1}{\pi} \int_{0}^{s_0} \frac{\text{Im} F_{\gamma^*\gamma^*\pi}^{PT}(Q^2, s)}{m_\rho^2 + q^2} e^{(m_\rho^2 - s)/M^2} ds + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im} F_{\gamma^*\gamma^*\pi}^{PT}(Q^2, s)}{s + q^2} ds$$

$s_0 \approx 1.5 \text{ GeV}^2$ – effective threshold in vector channel, $M^2$ – Borel parameter (0.5 – 0.9 GeV$^2$).

Real-photon limit $q^2 \rightarrow 0$ can be easily done ...
Khodjamirian [EJPC (1999)]: LCSR effectively accounts for long-distances effects of real photon using quark-hadron duality in vector channel and dispersion relation in $q^2$

$$F_{\gamma^*\gamma^*\pi}(Q^2, 0) = \frac{1}{\pi} \int_0^{s_0} \frac{\text{Im} F^{\text{PT}_{\gamma^*\gamma^*\pi}}(Q^2, s)}{m^2_\rho} e^{(m^2_\rho - s)/M^2} ds$$

$$+ \frac{1}{\pi} \int_{s_0}^\infty \frac{\text{Im} F^{\text{PT}_{\gamma^*\gamma^*\pi}}(Q^2, s)}{s} ds$$

$s_0 \simeq 1.5 \text{ GeV}^2$ – effective threshold in vector channel,
$M^2$ – Borel parameter ($0.5 - 0.9 \text{ GeV}^2$).

... as demonstrated here.
Main Ingredients of Spectral Density

We denote
\[ \rho(Q^2, s) = \rho^{(0)}(Q^2, s) + a_s \rho^{(1)}(Q^2, s) + a_s^2 \rho^{(2)}(Q^2, s) \]

- **NLO** Spectral Density — in [Mikhailov&Stefanis(2009)], partially corrected in [ABOP(2011)]:
  \[ \rho^{(1)}(Q^2, s) = \frac{\text{Im}}{\pi} \left[ (T_1 \otimes \varphi_\pi)(Q^2, -s - i\varepsilon) \right], s \geq 0 \]

- **NNLO** Spectral Density — in [M&S(2009)]
  \[ \rho^{(2,\beta)}(Q^2, s) = \beta_0 \frac{\text{Im}}{\pi} \left[ (T_{2\beta} \otimes \varphi_\pi)(Q^2, -s - i\varepsilon) \right], s \geq 0 \]
  Both \( \rho^{(1)} \) and \( \rho^{(2,\beta)} \) are obtained for arbitrary Gegenbauer harmonic.

- "Tw-6" contribution — in [ABOP–PRD83(2011)0540020]
  \[ \rho^{\text{tw6}}(Q^2, x) = 8\pi C_F \frac{\alpha_s \langle \bar{q}q \rangle^2 x^2}{N_c f^2_\pi} \frac{\alpha_s}{Q^6} \left[ 2x \ln x \bar{x} - x + 2\delta(\bar{x}) - \left[ \frac{1}{1-x} \right] \right] \]
High order corrections result

Twist-6 and NNLO$_{\beta_0}$ contributions to the $Q^2 F_{\gamma^* \gamma \pi}(Q^2)$ with BMS-like Pion DA

They practically cancel out each other [BMPS(2011)]

We use this residual as theoretical uncertainty of our prediction, that provides us with an additional 3%-uncertainty.
Pie chart for Pion-Photon TFF at $Q^2 = 8 \text{ GeV}^2$

Result is dominated by Hard Part of Twist-2 LO contribution.

Blue = negative terms
Red = positive terms

$F_{\pi\gamma}(Q^2)$
Pie chart for Pion-Photon TFF at $Q^2 = 8$ GeV$^2$

- Result is dominated by Hard Part of Twist-2 LO contribution.
- Twist-6 contribution is taken into account together with NNLO $\beta_0$ one — they have close absolute values and opposite signs.

Blue = negative terms
Red = positive terms
Parameters of LCSRs

From PDG:
- $\alpha_s(m_Z^2)$
- Masses $m_\rho, m_\omega$
- Decay Widths $\Gamma_\rho, \Gamma_\omega$

From QCD SR:
- Borel parameter $M_{LCSR}^2$
- Vector Chan. Threshold $s_0$
- Twist-4 $\delta^2 \pm 20\%$
- Twist-6 ($\alpha_s\langle\bar{q}q\rangle$)

Light-Cone Sum Rules:
$$\text{LO + NLO + Tw-4 + (NNLO}_{\beta_0} + \text{Tw-6)}$$

- $\pi$-DA model
- FF Prediction
- Fitting $\pi$-DA ($a_n$)
- Data on FF
LCSR Results for Pion-Gamma Transition FF
Comparison with all data: CELLO, CLEO and BaBar

\[ Q^2 F(Q^2) \ [\text{GeV}^2] \]

\[ Q^2 \ [\text{GeV}^2] \]

- BMS bunch describes very good all data for \( Q^2 \leq 9 \ \text{GeV}^2 \).
Comparison with all data: CELLO, CLEO and BaBar

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- Note added BaBar \( \gamma^*\gamma \to \eta, \eta' \) and \( e^+e^- \to \gamma\eta, \gamma\eta' \) data (1101.1142[hep-ex]): All they are inside BMS strip!
Comparison with all data: CELLO, CLEO and BaBar

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- Note added BaBar \( \gamma^*\gamma \rightarrow \eta, \eta' \) and \( e^+e^- \rightarrow \gamma\eta, \gamma\eta' \) data (1101.1142[hep-ex]): All they are inside BMS strip!
- **ABOP models** are in between two sets of BaBar data.

}\[ Q^2 \, [\text{GeV}^2] \]
Fitting Pion DA

Confidential Regions
How many harmonics take into account?

The goodness-of-fit $\chi^2_{\text{ndf}}$ criterium as a function on number $n$ of fit parameters.

1 - 9 Gev$^2$ data region

- Goodness is stable, while the error grows with $n$. The compromise — at $n = 2$ or $3$ with $\chi^2_{\text{ndf}} \approx 0.5$. 
How many harmonics take into account?

The goodness-of-fit $\chi^2_{\text{ndf}}$ criterium as a function on number $n$ of fit parameters.

- **1 - 9 Gev$^2$** data region

- **1 - 40 Gev$^2$** data region

- Goodness is stable, while the error grows with $n$. The compromise — at $n = 2$ or $3$ with $\chi^2_{\text{ndf}} \approx 0.5$.

- For fitting **1 - 40 Gev$^2$** data region one should take $n \geq 3$ parameters.
We fitted experimental data on $\pi\gamma$ TFF by varying Gegenbauer coefficients of Pion DA.

Two sets of experim. data (1 – 9 GeV$^2$ & 1 – 40 GeV$^2$) were analyzed to show the influence of BaBar Data on Pion DA.

Fit based on LCSRs with NLO+Tw4+3 Gegenbauers
**NLC SR Results vs 3D Constraints**

BMPS [PRD84(2011)034014]: 3D $1\sigma$-error ellipsoid at $\mu_{SY} = 2.4$ GeV scale without $\Delta \delta_{tw4}^2$ uncertainty

- Data Set $1 - 9 \text{ GeV}^2$
  - $\Leftrightarrow$ 2D projection of $1\sigma$-error ellipsoid
  - $\nabla \Leftrightarrow \chi_{ndf}^2 \approx 0.4$
  - $\times \Leftrightarrow$ BMS model with $\chi_{ndf}^2 \approx 0.5$
  - Best-fit $= (0.17, -0.14, 0.12 \pm 0.14)$
  - BMS $= (0.14, -0.09)$

**Good agreement** of all data at $Q^2 \leq 9 \text{ GeV}^2$

At 68.3\% CL we have good intersection $2D \cap 3D \cap 4D \neq \emptyset$
**NLC SR Results vs 3D Constraints**

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---

**Data Set 1 − 9 GeV$^2$**

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- $\nabla \leftrightarrow \chi^2_{ndf} \approx 0.4$
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**NLC SR Results vs 3D Constraints**

**BMPS [PRD84(2011)034014]:** 3D $1\sigma$-error ellipsoid at $\mu_{SY} = 2.4$ GeV scale without $\Delta \delta_{tw4}^2$ uncertainty

---

**Data Set $1 - 40$ GeV$^2$**

- Red $\Leftrightarrow$ 2D projection of $1\sigma$-error ellipsoid
- ▼ $\Leftrightarrow$ $\chi^2_{ndf} \approx 1.0$
- ★ $\Leftrightarrow$ BMS model with $\chi^2_{ndf} \approx 3.1$

**Best-fit** = $(0.18, -0.17, 0.31 \pm 0.1)$

**BMS** = $(0.14, -0.09)$

**Bad agreement** of all data at $Q^2 \leq 40$ GeV$^2$

At $68.3\%$ CL we have no intersection $2D \cap 3D = \emptyset$, $3D \cap 4D = \emptyset$. 

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Dual Year Russia–Spain@Barcelona (Spain), Nov. 8–11, 2011

πDA and πFFs in QCD SRs – p. 30
NLC-bunch and lattice prediction at $\mu_{SY} = 2.4$ GeV scale with accounting for $\Delta \delta_{tw4}^2$ uncertainty.

DAs: ◆ $\Leftrightarrow$ Asymp., ▲ $\Leftrightarrow$ ABOP-3, ✖ $\Leftrightarrow$ BMS, ■ $\Leftrightarrow$ CZ

Lattice’10 estimate of $a_2$ are shown by vertical lines.

BMS bunch agrees well with the lattice data

Data Set $1 - 9$ GeV$^2$
2D-Analysis of the data at $\mu_{SY} = 2.4$ GeV scale with accounting for $\Delta \delta_{tw4}^2$ uncertainty.

DAs: $\bullet \iff$ Asymp., $\blacksquare \iff$ ABOP-3, $\times \iff$ BMS, $\blacksquare \iff$ CZ

Lattice’10 estimate of $a_2$ are shown by vertical lines.

BMS bunch agrees well with the lattice data
BMS bunch has better agreement with data up $9 \text{ GeV}^2$ than with CLEO data only.
2D-Analysis of the data at $\mu_{SY} = 2.4$ GeV scale with accounting for $\Delta \delta_{tw4}^2$ uncertainty.

DAs: ◆ ⇔ Asymp., ▲ ⇔ ABOP-3, ✹ ⇔ BMS, ■ ⇔ CZ

Lattice’10 estimate of $a_2$ are shown by vertical lines.

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BMS bunch has better agreement with data up to $9 \text{ GeV}^2$ than with CLEO data only.
NLC SR Results vs 2D Constraints

BMPS [arXiv:1105.2753 [hep-ph]]: 2D $1\sigma$-error ellipses at $\mu_{SY} = 2.4$ GeV scale with accounting for $\Delta \delta_{tw4}^2$ uncertainty.

DAs: $\blacklozenge \Leftrightarrow$ Asymp., $\blacktriangle \Leftrightarrow$ ABOP-3, $\blacklozenge \Leftrightarrow$ BMS, $\blacksquare \Leftrightarrow$ CZ

Lattice’10 estimate of $a_2$ are shown by vertical lines.

Data Set $1 - 40 \text{ GeV}^2$

- $\Leftrightarrow$ 2D $1\sigma$-error ellipse
- $\Leftrightarrow$ 2D-Proj. 3D-ellipsoid

Bad agreement with 2D $1\sigma$-error ellipse
No cross-section with $a_6 = 0$ plane.
**3D Data Fit of Pion DA vs BMS (QCD SR)**

- **:= BMS, := 1 − 9 GeV², := 1 − 40 GeV²**

at $\mu_{SY} = 2.4$ GeV scale.

- **BMS bunch agrees well with Data Set $1 − 9$ GeV²;**
- **New BaBar Data do not agree with BMS bunch based on NLC QCD SRs.**
- **Both data sets does not match each other.**
Integral derivative \( D^{(2)} \varphi(x) = \frac{1}{x} \int_{0}^{x} \frac{\varphi(y)}{y} dy \)

is an average derivative \( \varphi'_\pi(x) \) near the end-point \( x = 0 \).

Important property: \( \lim_{x \to 0} D^{(2)} \varphi(x) = \varphi'_\pi(0) \).
Integral derivative $D^{(2)} \varphi(x) = \frac{1}{x} \int_0^x \frac{\varphi(y)}{y} dy$

at $\mu_{SY} = 2.4$ GeV scale.

$\Phi_{\pi}$ and $\varphi$ vs. $x$ for $1 \leq x \leq 40$ GeV$^2$

$\Phi_{\pi}$ and $\varphi$ vs. $x$ for $1 \leq x \leq 9$ GeV$^2$

$\bullet$ $DA^{1-9}$ GeV$^2$ and $DA^{1-40}$ GeV$^2$ are separated near the origin.

$\bullet$ BaBar Data demands End-Point Enhanced Pion DA.
Fit Results and Pion DA Models
### Comparing Fit Results with Pion DA models

<table>
<thead>
<tr>
<th>Model/Fit</th>
<th>Values of $a_n$</th>
<th>$\chi^2$/ndf (1 − 9 GeV$^2$)</th>
<th>$\chi^2$/ndf (1 − 40 GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_2$, $a_4$, $a_6$ fit</td>
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**BMS DA** gives best LCSR Description of $\pi\gamma$ TFF for $Q^2 \leq 9 \text{ GeV}^2$. 
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- **All-Data** LCSR-Fit Result is far from All Considered Pion DA Models.

Dual Year Russia–Spain@Barcelona (Spain), Nov. 8–11, 2011

$\pi$DA and $\pi$FFs in QCD SRs – p. 36
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1σ region in \((\langle \xi^2 \rangle_\pi, \langle \xi^4 \rangle_\pi)\) plane from 2D\((1 - 9\ \text{GeV}^2)\) analysis vs QCDSF&UKQCD Lattice Data [PRD74(2006)074501] at \(\mu_{\text{lat}} = 2\ \text{GeV}\) scale:

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Our 2D-1σ region is almost completely inside Lattice’06 constraint.
$1\sigma$ region in $(\langle \xi^2 \rangle_\pi, \langle \xi^4 \rangle_\pi)$ plane from $2D(1 - 9 \text{ GeV}^2)$ analysis vs RBC&UKQCD Lattice Data [PRD83(2011)074505] at $\mu_{\text{lat}} = 2 \text{ GeV}$ scale:

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Our $2D-1\sigma$ region is one-half inside Lattice’10 constraint.
1σ region in \((\langle \xi^2 \rangle_{\pi}, \langle \xi^4 \rangle_{\pi})\) plane from 2D\((1 - 9 \text{ GeV}^2)\) analysis vs RBC&UKQCD Lattice Data [PRD83(2011)074505] at \(\mu_{\text{lat}} = 2 \text{ GeV}\) scale:

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Our 2D-1σ region with \((M^2 \approx 0.7 \text{ GeV}^2)\) is one-half inside Lattice’10 constraint,

whereas the 2D-1σ region with ABOP value \((M^2 = 1.5 \text{ GeV}^2)\) is completely out of Lattice’10 constraint!
$1\sigma$ region in $(\langle \xi^2 \rangle_\pi, \langle \xi^4 \rangle_\pi)$ plane from $2D(1 - 9 \text{ GeV}^2)$ analysis vs RBC&UKQCD Lattice Data [PRD83(2011)074505] at $\mu_{\text{lat}} = 2 \text{ GeV}$ scale:

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Intersection of Lattice and 2D-1$\sigma$ region leads to prediction:

$\langle \xi^4 \rangle_\pi \in [0.11, 0.122]$ — in a good agreement with estimation $\langle \xi^4 \rangle_\pi \in [0.095, 0.134]$ in [Stefanis, NPB.PS.181(2008)199].
Pion DA
and
Drell–Yan $\pi N$-process
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Brandenburg et al. [PRL (1994)]: Unpolarized case
Brandenburg et al. [PRD (1996)]: Polarized case

As $x_u \rightarrow 1$, $p_{u}^2$ becomes large and far spacelike and it is sufficient to consider $u$ quark to be nearly free and on-shell: $x_u = x_N$ (no transverse momenta).

$q^2 = 4 - 81 \text{ GeV}^2$
$s = 100 - 400 \text{ GeV}^2$
Angular distribution parameters

$\theta$: Polar angle measuring $\mu^+$ direction in Gottfried–Jackson system of axes.

$\phi$: Azimuthal angle between $\pi^- \mu^+ \mu^-$ and $\pi^- N$ planes in lepton rest frame.
Angular distribution parameters

Cross-section:

\[
\frac{d^5 \sigma (\pi^- + N^{\text{pol}}(s_l) \rightarrow \mu^+ + \mu^- + X)}{dQ^2 dQ_T^2 dx_L d\cos \theta d\phi} \propto 1 + \lambda \cos^2 \theta + (\mu + \tilde{\mu} s_l) \sin 2\theta \cos \phi + \frac{(\nu + \tilde{\nu} s_l)}{2} \sin^2 \theta \cos 2\phi
\]
Asymmetry $\mathcal{A}$ of polarized DY $\pi^- N$ process

Following convolution BMT procedure, we found at $\rho = 0.3$, $s = 100 \text{ GeV}^2$, and $Q^2 = 16 \text{ GeV}^2$:

Asymmetry 3D plot for Asymptotic DA
Asymmetry $\mathcal{A}$ of polarized DY $\pi^- N$ process

Following convolution BMT procedure, we found at $\rho = 0.3$, $s = 100 \text{ GeV}^2$, and $Q^2 = 16 \text{ GeV}^2$:

Asymmetry 3D plot for BMS DA
Asymmetry $\mathcal{A}$ of polarized DY $\pi^- N$ process

Following convolution BMT procedure, we found at $\rho = 0.3$, $s = 100 \text{ GeV}^2$, and $Q^2 = 16 \text{ GeV}^2$:

Asymmetry 3D plot for CZ DA
Concluding: Asymmetry $\mathcal{A}(\phi, x_L)$ can be used to discriminate different pion DA models if possible to fix the value of $\rho$ in the experiment.
Asymmetry $\mathcal{A}$ of polarized DY $\pi^- N$ process

**Conclusion:** Asymmetry $\mathcal{A}(\phi, x_L)$ can be used to discriminate different pion DA models if possible to fix the value of $\rho$ in the experiment.

Task for the COMPASS experiment!
Conclusions

Result of fitting the CELLO, CLEO, and BaBar Data up to $9 \text{ GeV}^2$ is in a good agreement with previous CLEO-based fit and prefers the End-Point Suppressed (BMS) Pion DA.
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Year 2003: JINR – Santiago de Compostela University
1st prize of “Physics of Particles and Nuclei” journal for Kataev–Parente–Sidorov paper
Dubna – Spain Collaboration

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- **Years 2012–13:** May be it is a good time to continue?
Gracias por su atención!
Unpolarized Drell–Yan $\pi^- N$ process: $\lambda(x, \rho)$

Following convolution procedure of Brandenburg–Müller–Teryaev, we found ($\rho = Q_T/Q$):

Agreement of pion DA model with unpolarized E615 (FNAL) data depends on the value of $\rho$. 

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$\rho = 0.3$
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