Structure and production of medium-mass hypernuclei

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(Osaka E-C)
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Talk is based mostly on
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4. Propose to use odd-Z targets available in medium-mass region
5. Summay
1. Introduction

Why medium-mass hypernuclei?

Basic motivation (1):

The great success of JLab Hall A and Hall C experiments:

-- sub-MeV resolution ($\Gamma = 0.5$ MeV)

-- $p$-shell theor. predictions: confirmed

These facts encourage extension of high-resolution reaction spectroscopy to heavier hypernuclei:
played a great role of exciting high-spin series $\Gamma = 1.5 \text{ MeV (best)}$

Motoba. Sotona, Itonaga,

JLab Exp’t : $\Gamma = 0.5 \text{ MeV}$
Theor. prediction vs. (e,e’K^+) experiments


------------ Sotona’s Calc.----→
Hall C (up) T. Miyoshi et al.
P.R.L. **90** (2003) 232502. \(\Gamma=0.75\) MeV
Hall A (bottom), J.J. LeRose et al.
N.P. A **804** (2008) 116. \(\Gamma=0.67\) MeV
Why medium-mass hypernuclei?

Basic motivation (2)

Unique characteristics of the \((e,e'K^+)\), \((\gamma, K^+)\) process are based on the basic properties of elementary amplitudes for \(\gamma p \rightarrow \Lambda K^+\):

--- **sizable momentum transfer** to excite high-spin states, like \((\pi^+, K^+)\)

--- **spin-flip dominance** of the operator, leading to unnatural parity states
2. A brief review of theoretical treatments for hypernuclear production cross sections

\[ A^Z(J_i, T_i, \tau_i)(K^-, \pi^-)A^Z(J_f, T_f, \tau_f) \]  

\( (\pi^+, K^+) \)  
\( (\gamma, K^+) \)  
\( (K^-, K^+) \)

Three factors:
1. PW vs. DW (DW effects)
2. Microscopic treatment with elem. amplitudes,
3. Nuclear core excitation effects
\[(A) \text{ FACTORIZED VS. (B) MICROSCOPIC}\]

\[(A) \text{ Factorized DWIA treatment}\]

by Huefner-Lee-Weidenmuleler, NPA234, 429 (1974)

\[
\left( \frac{d\sigma(\theta)}{d\Omega_L} \right)^{(K^-,\pi^-)} = \alpha \left( \frac{d\bar{\sigma}(\theta)}{d\Omega_L} \right)^{K^−n\Lambda\pi^-} N_{\text{eff}}^{(K^-,\pi^-)}(i f; \theta)
\]

\[\alpha= \text{kinematical factor for A-body to 2-body transformation,}\]

\[N_{\text{eff}}= \text{Effective neutron number :}\]

\[
N_{\text{eff}}^{(K^-,\pi^-)}(i f; \theta) = \frac{1}{[J_i]} \sum_{M_iM_f} \left| \left\langle J_fM_fT_f\tau_f \right| \int dr \chi_\pi^{(-)*}(k_\pi, \frac{M_A}{M_H} r) \chi_\pi^{(+)}(k_K, r) \right|^2 \\
\times \sum_{j=1}^{A} U_-(j) \delta \left( r - \frac{M_c}{M_A} r_j \right) \left| J_iM_iT_i\tau_i \right|^2
\]
(A-1) Meson waves by the Eikonal approximation

\[ N_{\text{eff}}(Z_{\text{eff}}) = \int \rho_{n(p)}(r) \cdot \exp \left[ -\bar{\sigma}_{aN} \int_{-\infty}^{Z} \rho(x, y, z') dz' \right. \]
\[ \left. -\bar{\sigma}_{bN} \int_{Z}^{\infty} \rho(x, y, z') dz' \right] d^3r \]

Applicable to forward scattering, 10-20% error of K-G DW (Auerbach et al. (1983)).

(A-2) Meson DW with the Klein-Gordon solutions

\[ N_{\text{eff}} = \frac{A}{2J + 1} \sum_{M_i M_f} \left| \int \Psi_f^*(A Z) \hat{O}(r) \Phi_i(A Z) \prod_{i=1}^{A} dr_i \right|^2, \]

\[ \hat{O}_{ab}(\theta; r) = \int d^3r \chi_{p_b}^{(-)*}(r) \cdot \chi_{p_a}^{(+)}(r) \sum_{i=1}^{A} \hat{X}(i) \delta(r - r_i) \]

\[ \chi_{p_b}^{(-)*}(r) \cdot \chi_{p_a}^{(+)}(r) = \sum_{\kappa} \sqrt{4\pi[\kappa]} i^\kappa \tilde{j}_\kappa(p_b, p_a, \theta; r) Y_\kappa(\hat{r}_{ab}). \]
PW vs. DW

(1) DW effect
In a typical $(\pi^+, K^+)$:
$N_{\text{eff}} = 0.184$ (PW)
$0.030$ (DW)

(2) XS to low-J states are much more reduced, resulting in the sharper peaks

(3) Low-$L$ partial waves are reduced by distortion
(B) Microscopic treatment based on the elementary transition amplitudes \((\pi,K)\) case

\[
\frac{d\sigma(\theta_L)}{d\Omega_L} = \gamma \cdot \frac{(2\pi)^4 p_K^2 E_\pi E_K E_H}{p_\pi \{p_K(E_H + E_K) - p_\pi E_K \cos\theta_L\}} \overline{|T_{if}^L|^2},
\]

\[
|T_{if}^L|^2 = \sum_{M_f} R(if; M_f),
\]

\[
R(if; M_f) = \frac{1}{[J_i]} \sum_{M_i} \left| \langle J_f M_f | \int d^3r \chi^{(-)}(p_K; r)^* \cdot \chi^{(+)}(p_\pi; r) \right| \\
\times \sum_{k=1}^{A} U_-(k) \delta(r - r_k) \cdot \lambda [f + ig(\sigma_k \cdot \hat{n})] |J_i M_i| \right|^2,
\]

Elementary amplitude \(N \rightarrow Y\)

\(f = \text{spin-nonflip}, \ g = \text{spin-flip}, \ \sigma = \text{baryon spin}\)
M. Sotona and J. Zofka described the $\pi n \rightarrow \Lambda K$ cross sections and polarization data in terms of $f$ and $g$ elementary amplitudes so as to be easily applied to hypernuclear production.

J. Zofka stayed at INS where O. Hashimoto was preparing the (p+,K+) experiment.
$R(i,f;M)$ is expressed with three kinds of the reduced effective numbers (microscopic)

$$R(if;M_f) = \chi^2 \left\{ |f|^2 \rho^{ff}(if;M_f) + |g|^2 \rho^{gg}(if;M_f) + 2 \text{Im}[fg^*] \rho^{fg}(if;M_f) \right\}$$

Magnetic subspace population $P(i,f : M)$ is defined by

$$P(if;M_f) = \frac{R(if;M_f)}{\sum_{M_f} R(if;M_f)}$$

Polarization of Hypernuclear state $|J_f>$ is calculated by

$$\mathcal{P}(J_f) = \sum_{M_f} M_f P(if;M_f) / J_f.$$
3. Electro/photo-production of sd-shell hypernuclei

- Microscopic cal. based on elem. ampl.
- DW: solution of the Klein-Gordon eq.
- Emphasize the importance of taking account of nuclear core excitation effects
Hyperon recoil momentum and the transition operator determine the reaction characteristics.

$q_\Lambda = 350-420$ MeV/c at $E_\gamma = 1.3$ GeV
Lab $d\sigma/d\Omega$ for photoproduction (2Lab)

$$
\frac{d\sigma}{d\Omega} \bigg|_{2\text{Lab}} = \frac{(2\pi)^4 p^2 E_K E_L}{k(p(E_A + E_K) - kE_K \cos \theta_L)} \left| \langle k - p, p | t | k, 0 \rangle_L \right|^2 ,
$$

(2.4)

$$
\langle k - p, p | t | k, 0 \rangle_L = a_1(\sigma \cdot \epsilon) + a_2(\sigma \cdot \hat{k})(\hat{p} \cdot \epsilon) + a_3(\sigma \cdot \hat{p})(\hat{p} \cdot \epsilon) + a_4((\hat{k} \times \hat{p}) \cdot \epsilon).
$$

(2.5)

Spin non-flip term

$$
\langle k - p, p | t | k, 0 \rangle_L = \epsilon_0(f_0 + g_0 \sigma_0) + \epsilon_x(g_1 \sigma_1 + g_{-1} \sigma_{-1})
$$

(2.11)

with definitions of the coefficients:

$$
f_0 = a_4 \sin \theta_L ,
$$

$$
g_0 = a_1 ,
$$

$$
g_{\pm 1} = \frac{1}{\sqrt{2}} \{ \mp (a_1 + a_3 \sin^2 \theta_L) - i \sin \theta_L (a_2 + a_3 \cos \theta_L) \}.
$$

(2.12)
These characteristic merits of the $\gamma p \rightarrow \Lambda K^+$ process (ability to excite high-spin unnatural-parity states) should be realized better in heavier systems involving large $j_p$ and large $j_\Lambda$

\[(e,e'K^+) \quad \frac{d^3\sigma}{dE_e d\Omega_e d\Omega_K} = \Gamma \times \frac{d\sigma}{d\Omega_K}\]

$\Gamma$ : virtual photon flux (kinematics)

Hereafter we discuss $\frac{d\sigma}{d\Omega_K}$ for $^A_Z(\gamma,K^+)_\Lambda^A_{Z'}$
3-1. The simplest sd-shell target

Choose the $^{19}\text{F}(1/2^+)$ target ($^{16}\text{O}+\text{p}+2\text{n}$) for demonstration of the hypernuclear photoproduction

(asking the feasibility as a practical target)
Choose $^{19}$F($1/2^+$) target for demonstration

Shell-model configuration
neutron   proton
Conversion of $1s_{1/2}$-proton (nb/sr)

Partial contributions

cf. A trial calculation: If the last odd proton were in $0d_{5/2}$, then
Partial contributions from core-excitation

Conversion of $0p_{1/2} \rightarrow \Lambda (s,p,d)$

Conversion from $1p_{3/2}$

(use arbitrary widths for proton $p_{1/2}$ and $p_{3/2}$.)
As a “closed core ($^{18}\text{O}$)+ $\Lambda$, cf. SO-splitting($0p$)=152+-54 keV(C13)
Exp. $C^2S$ for p-pickup from $^{16}\text{O}$ core

$$
\begin{align*}
1S_{1/2} &\quad 0.871 \quad (0.21) \quad 1/2^+ \\
0d_{5/2} &\quad 0.0 \quad (1.53) \quad 5/2^+ \\
0d_{3/2} &\quad 5.085 \quad (0.10) \quad 3/2^+ \\
1S_{1/2} &\quad 0.871 \quad (0.21) \quad 1/2^+ \\
0d_{5/2} &\quad 0.0 \quad (1.53) \quad 5/2^+ \\
1S_{1/2} &\quad 0.871 \quad (0.21) \quad 1/2^+ \\
0d_{5/2} &\quad 0.0 \quad (1.53) \quad 5/2^+ \\
\end{align*}
$$

$^{18}\text{O}(d,t)^{17}\text{O}$ ($C^2S$)

Mairle et al. N.P. A280 (1878) 97.
Lightest sd-shell target: $^{19}\text{F}$

A’s: $p_{1/2}$-hole series, B’s: $p_{3/2}$-hole series

$^{19}\text{F}(\gamma,K^+)^{19}\Lambda\text{O}$ $E_\gamma = 1.3$ GeV

Hypernuclear Excitation Energy $E_\Lambda$ (MeV)
Umeya’s Calculation

Results: Production cross sections of $^{19}\text{F}(K^-,\pi^-)$ and $^{19}\text{F}(\pi^+,K^+)$

$^{19}\text{F}(K^-,\pi^-)\ ^{19}\text{F}_\text{Λ}$

$(p_K = 0.80 \text{ GeV}/c, \theta_{\text{Lab}} = 3^\circ)$

$^{19}\text{F}(\pi^+,K^+)\ ^{19}\text{F}_\text{Λ}$

$(p_\pi = 1.05 \text{ GeV}/c, \theta_{\text{Lab}} = 3^\circ)$
Umeya’s Calculation

Results: Low-lying energy levels of $^{19}_{\Lambda}$F and $^{18}$F

The present shell model should be improved in order to describe the energies of the negative-parity states of $^{18}$F.
A typical example of medium-heavy target: $^{28}\text{Si}$: assuming $(d_{5/2})^6$ closure.

to show characteristics of the $(\gamma,K^+)$ reaction with DDHF w.f.

( Spin-orbit splitting: consistent with $^7\text{Li}, ^9\text{Be}, ^{13}\text{C}, ^{89}\text{Y}$ )
Theor. x-section for \((d_{5/2})^6 (\gamma, K^+)\) \([j_h-j_\Lambda]J\)

<table>
<thead>
<tr>
<th>DWIA</th>
<th>(\lambda=\frac{1}{2}L)</th>
<th>p3/2L</th>
<th>p1/2L</th>
<th>1s1/2L</th>
<th>d5/2L</th>
<th>d3/2L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>((-16.92))</td>
<td>((-8.40))</td>
<td>((-8.40))</td>
<td>((0.32))</td>
<td>((0.69))</td>
<td>((0.69))</td>
</tr>
<tr>
<td>Proton hole</td>
<td>1- 5.4 2- 7.1 2- 19.4 2+ 2.2</td>
<td>0+ 0.0 1+ 26.0 1+ 8.9</td>
<td>2+ 0.3 2+ 34.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d(\frac{5}{2})</td>
<td>3+ 63.8 4- 141.8</td>
<td>3+ 76.2 3+ 4.6</td>
<td>3+ 26.7 3+ 30.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g.s.)</td>
<td>2+ 29.2 1- 30.5 1+ 2.0 2+ 66.9</td>
<td>1+ 28.3 1- 12.2 2- 10.7 2- 43.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(\frac{1}{2})</td>
<td>0- 9.4 1- 30.5 1+ 2.0 2+ 66.9</td>
<td>0+ 0.0 0- 3.7 1- 1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((-25.49))</td>
<td></td>
<td></td>
<td>3- 76.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p(\frac{3}{2})</td>
<td>1- 14.3 1+ 8.9 1+ 1.8 1- 5.9 1- 3.2 0- 2.0</td>
<td>0- 2.0 1+ 5.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((-29.84))</td>
<td>2- 59.1 2+ 0.4 2+ 62.5 2- 24.8 2- 4.5 2+ 17.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3+ 109.1 3- 4.5 3+ 96.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s(\frac{1}{2})</td>
<td>0+ 0.1 1+ 19.2 1- 12.1 1- 23.7 1+ 51.4 2+ 27.0 2+ 40.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((-44.55))</td>
<td>2- 50.0 1+ 16.5 3+ 58.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
3-3. Realistic prediction for $^{28}\text{Si} \ (\gamma,K^+) \ \Lambda^{28}\text{Al}$ compared with exp.

By fully taking account of

-- full $p(sd)^6.n(sd)^6$ configurations,
-- fragmentations when a proton is converted,
-- $^{27}\text{Al}$ core nuclear excitation
-- $K^+$ wave distortion effects

→ Comparison with the $^{28}\text{Si} \ (e,e^{'K^+})$ exp.
proton-state **fragmentations** should be taken into account *to be realistic*
Proton pickup from $^{28}\text{Si}(0^+):(sd)^6 = (d_{5/2})^{4.1}(1s_{1/2})^{0.9}(d_{3/2})^{1.0}$
Peaks can be classified by the characters

\[ ^{28}\text{Si}(\gamma, K^+) \! _{\Lambda}^{28}\text{Al} \quad E_\gamma = 1.3 \text{ GeV} \quad \theta_K = 3^\circ \]

Hypernuclear Energy \( E_\Lambda \) (MeV)

Major peak series: \([ ^{27}\text{Al}(5/2^+_2, 3/2^+_2) \times j^\Lambda ]_J \) with \( j^\Lambda = s, p, d, \ldots \)
Peak energies: $^{28}_{\Lambda}Si$ vs. $^{28}_{\Lambda}Al$


<table>
<thead>
<tr>
<th>j (\Lambda)</th>
<th>$^{28}<em>{\Lambda}Si(p^+,K^+)^{28}</em>{\Lambda}Si$ $E_{\Lambda}=-B_{\Lambda}$ (Ex)</th>
<th>$^{28}<em>{\Lambda}Si(e,e'K^+)^{28}</em>{\Lambda}Al$ (as read on the Sendai08 poster)</th>
<th>$(\gamma,K^+)$ CAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>-16.6+-0.2 (GS)</td>
<td>-17.85 ?? (GS)</td>
<td>-16.6 (GS)</td>
</tr>
<tr>
<td>p</td>
<td>-7.0+-0.2 ($E_x=9.6$)</td>
<td>-6.88 +- ?? ($E_x=11$)</td>
<td>-8.1 ($E_x=8.5$)</td>
</tr>
<tr>
<td>d</td>
<td>+1.0+-0.8 ($E_x=17.6$)</td>
<td>+1.35 +/- ($E_x=19.2$)</td>
<td>+0.9 ($E_x=17.5$)</td>
</tr>
</tbody>
</table>
3-4. Extend to heavier nuclear targets

$^{52}\text{Cr}: (f_{7/2})^4$ assumed

$^{40}\text{Ca}: (sd\text{-shell LS-closed})$
52Cr (j≥ dominant target case)
typical unnatural-parity high-spin states

\[ 52\text{Cr}(\gamma,K^+)^{52}_{Λ}V \quad E_γ = 1.3 \text{ GeV} \quad θ_K = 3° \]

**Hypernuclear Energy**  \[ d^2σ/dΩdE \]  \[ E_Λ \text{ (MeV)} \]  

**Major peak series**: \[ 51V(7/2^+; gs) × j^Λ \] with \( j^Λ = s, p, d, f, ... \)
Well-separated series of peaks due to large $q$ and spin-flip dominance:

$j_\uparrow = l + 1/2$, $j_\downarrow = l - 1/2$

\[
[(nlj)^{-1}_p (nlj)^\Lambda_j]_J \quad \text{a series of pronounced peaks}
\]

$jj$-closed target: ($^{28}\text{Si}$, $^{52}\text{Cr}$)

\[
[J = j_\uparrow + j_\downarrow^\Lambda = l_p + l_\Lambda + 1 = L_{\max} + 1 \quad \text{(unnatural parity)}]
\]

$LS$-closed target: ($^{40}\text{Ca}$)

\[
[J = j_\downarrow + j_\uparrow^\Lambda = l_p + l_\Lambda = L_{\max} \quad \text{(natural parity)}]
\]
$^{40}\text{Ca}$ (LS-closed shell case): high-spin states with natural-parity $(2^+, 3^-, 4^+)$

Major peak series: $\left[ ^{39}\text{K}(d_{3/2}^{-1}; gs) \times j^\Lambda \right]_J$ with $j^\Lambda = s, p, d, f, \ldots$
4. Propose to use odd-proton targets in sd- and fp-shell regions: \( \Lambda \) energies within sub-MeV resolution

\( \Lambda \) hyperon on the 0+ core

\( \Lambda \) coupled with core-excited configuration
Single-particle energies of $\Lambda$

G-matrix results vs. experiments


$Sd$- and $fp$-shell data are quite important to extract the $\Lambda$ behavior in nuclear matter.
Available odd-Z targets

\begin{align*}
^{19}_{9}F_{10} & (100\%) \ [ J=1/2^+] : & ^{18}_{8}O_{10} + p(1s_{1/2}) \\
^{23}_{11}Na_{12} & (100\%) \ [ J=5/2^+] : & ^{22}_{10}Ne_{12} + p(0d_{5/2}) \\
^{27}_{13}Al_{14} & (100\%) \ [ J=5/2^+] : & ^{26}_{12}Mg_{14} + p(0d_{5/2}) \\
^{31}_{15}P_{16} & (100\%) \ [ J=1/2^+] : & ^{30}_{14}Si_{16} + p(1s_{1/2}) \\
^{39}_{19}K_{20} & (93.26\%) [ J=3/2^+] : & ^{38}_{18}Ar_{20} + p(0d_{3/2}) \\
^{45}_{21}Sc_{24} & (100\%) \ [ J=7/2^+] : & ^{44}_{20}Ca_{24} + p(0f_{7/2}) \\
^{51}_{23}V_{28} & (99.75\%) \ [ J=7/2^+] : & ^{50}_{22}Ti_{28} + p(0f_{7/2}) \\
^{55}_{25}Mn_{30} & (100\%) \ [ J=7/2^+] : & ^{54}_{24}Cr_{30} + p(0f_{7/2}) \\
\end{align*}

( feasibility to be checked by experimentalists )
1) Based on the elementary amplitudes, the microscopic theoretical framework for hypernuclear production XS were discussed.

2) Several photo-production spectra have been calculated by taking account of major core-excitation effects. The prediction for $^{28}_{\Lambda}$Al is well compared with the recent experiments.

3) Predictions are made also for heavier typical targets, $^{40}$Ca and $^{52}$Cr, showing fruitful aspects.
4) Medium-mass hypernuclear production-by \((e,e’K^+)\) provide us with good opportunities in understanding the details of the hyperon motion in nuclear matter.

(\(\Lambda\)-s.p.e. to establish “textbook”, Rotation/Vib.-\(\Lambda\) coupling, Auger effect, \(\mu_\Lambda\), \(e_{\text{eff}}(\Lambda)\), etc)

Remark:

The present frameworks apply also to \(\Xi\)-hypernuclear production with sd-shell targets which might be fruitful at J-PARC.