Alpha–particle condensation in light hypernuclei

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XI International Conference on Hypernuclear and Strange Particle Physics (HYP2012) @ Barcelona, October 1 - 5, 2012
Motivation and purpose

In light $\Lambda$ hypernuclei

- glue-like role of $\Lambda$
  \[ \alpha + x + \Lambda (x=p,n,d,t,^3\text{He},\alpha), \quad ^{13}\Lambda\text{C} (3\alpha + \Lambda) \]
- Shrinkage of core nucleus
  \[ ^{20}\Lambda\text{Ne} (^{15}\text{O} + \alpha + \Lambda), \quad ^{21}\Lambda\text{Ne} (^{16}\text{O} + \alpha + \Lambda) \]

\[ \rightarrow \text{Reduction of } B(\text{E2}) \quad \text{Smaller moment of inertia?} \]

observed in $^7\Lambda\text{Li}$

today’s talk by H. Tamura

Motivation and purpose

In light $\Lambda$ hypernuclei

- glue-like role of $\Lambda$ $\quad \alpha + x + \Lambda \ (x=p,n,d,t,^3\text{He},\alpha)\ ), \ ^{13}_\Lambda\text{C} \ (3\alpha + \Lambda)$
- Shrinkage of core nucleus $\quad ^{20}_\Lambda\text{Ne} \ (^{15}\text{O} + \alpha + \Lambda), \ ^{21}_\Lambda\text{Ne} \ (^{16}\text{O} + \alpha + \Lambda)$

$\rightarrow$ Reduction of $B(E2)$ Smaller moment of inertia?
observed in $^{7}_\Lambda\text{Li}$ today’s talk by H. Tamura

In light nuclei

- gas-like cluster states exist.
- Called ``Alpha condensate’’,
  all boson clusters condensed into an identical lowest orbit.

Structural change when adding $\Lambda$
`gas phase' in finite nuclei

**Energy**

- **Excitation energy**
  - 7.2 MeV
  - 14.2 MeV
  - 19.2 MeV

- **nα threshold energy**
  - $\rho_0/3 \sim \rho_0/5$

**Infinite nuclear matter**

- (low density)
- $< \rho_0/5$

**Crust of neutron star?**

- **Cluster gas**

**Gas phase** in finite nuclei

- **Excitation**
- **Dissolution**

**$^{12}$C**

**$^{16}$O**

**$^{20}$Ne**

**Quantum liquid**

N $\alpha$ gas state + Λ

Condensates survive or broken?

Too much shrinkage → evidence of large size
First example of alpha cond. state

Cluster gas

\[ R_{\text{rms}} \approx 3.8 \text{ fm} \]

Condensed into the lowest orbit (lowest config. of gas)

\[ \alpha \text{ cluster} \]

\[ R_{\text{rms}} \approx 2.4 \text{ fm} \]

shell-model –like structure
Motivation and purpose

In light $\Lambda$ hypernuclei

- glue-like role of $\Lambda$ 
  $\alpha+x+\Lambda$ ($x=p,n,d,t,^3\text{He},\alpha$), $^{13}_{\Lambda}\text{C}$ ($3\alpha+\Lambda$)

- Shrinkage of core nucleus 
  $^{20}_{\Lambda}\text{Ne}$ ($^{15}\text{O}+\alpha+\Lambda$), $^{21}_{\Lambda}\text{Ne}$ ($^{16}\text{O}+\alpha+\Lambda$)

$\rightarrow$ Reduction of $\text{B(E2)}$ 
observed in $^7_{\Lambda}\text{Li}$

Smaller moment of inertia? 
today’s talk by H. Tamura


In light nuclei

- gas-like cluster states exist.

- Called “Alpha condensate”
  all boson clusters condensed into an identical lowest orbit.

  Structural change when adding $\Lambda$

- Mysterious $0^+$ states still exist.
Still mysterious $0^+$ state in $^{12}$C

Broad $0^+$ state at 10.3 MeV

- AMD: 3α linear-chain-like
- OCM+CSM: family of α cond. with broad width

No theory to predict the $0^+$ state consistently.

- Two $0^+$ states?

The structure is still now under question!

Invited talk by M. Itoh in Clusters12
Still mysterious $0^+$ state in $^{12}$C

Broad $0^+$ state at 10.3 MeV

- AMD: 3 $\alpha$ linear-chain-like
- OCM+CSM: family of $\alpha$ cond. with broad width

No theory to predict the $0^+$ state consistently.

- Two $0^+$ states?

The structure is still now under question!

Theoretically very difficult broad resonance state

\begin{align*}
\text{B(E2: } & 2^+_2 \rightarrow 0^+_3 \text{)} = 310 \text{ e}^2\text{fm}^4 \\
\text{B(E2: } & 2^+_2 \rightarrow 0^+_2 \text{)} = 100 \text{ e}^2\text{fm}^4
\end{align*}


Invited talk by M. Itoh in Clusters12
Still mysterious $0^+$ state in $^{12}$C

**Broad $0^+$ state at 10.3 MeV**

- **AMD**: 3 α linear-chain-like
- **OCM+CSM**: family of α cond. with broad width

No theory to predict the $0^+$ state consistently.

**Two $0^+$ states?**

The structure is still now under question!

Theoretically very difficult

**Broad resonance state**

Adding Λ particle, What happens?

Change to sharp resonance due to the glue-like role?

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8Be($0^+$)+α


B(E2: $2^+_2 \rightarrow 0^+_3$) = 310 e²fm⁴
B(E2: $2^+_2 \rightarrow 0^+_2$) = 100 e²fm⁴

---

Result of MDA
Ex = 9.04 ± 0.09 MeV
Γ = 1.45 ± 0.18 MeV

Ex = 10.56 ± 0.06 MeV
Γ = 1.42 ± 0.08 MeV


Invited talk by M. Itoh in Clusters12
Model

- $\alpha$ condensate type wave function (THSR)
- fully microscopic model
- only one parameter, $B$ (with deformation, $B_x, B_y, B_z$) which characterizes nuclear density

\[ \Phi_{3\alpha}^{THSR}(B) = A \]

$B \sim b$: ground state
$B \gg b$: $\alpha$ condensed state

Spatial shrinkage happens when $\Lambda$ particle is injected in a nucleus. The corresponding rearrangement effect can be optimally described.
Model

- $\alpha$ condensate type wave function (THSR)
- fully microscopic model
- only one parameter, $B$ (with deformation, $B_x, B_y, B_z$) which characterizes nuclear density

$$\Phi^{\text{THSR}}_{n\alpha} (r_1, \ldots, r_{4n}, B) = A \left\{ \prod_{i=1}^{n} e^{-\frac{2}{B^2}(x_i-x_G)^2} \phi_{\alpha_i} (r_{4i-3}, \ldots, r_{4i}, b) \right\}$$

\text{c.o.m. of $i$-th $\alpha$ particle}
$$X_i = \frac{r_{4i-3} + \cdots + r_{4i}}{4}$$

\text{Total c.o.m.}
$$X_G = \frac{r_1 + \cdots + r_{4n}}{4n}$$

With deformation, axially symmetric ($B_\perp, B_z$)
$$e^{-\frac{2}{B^2}(x-x_G)^2} \rightarrow \chi^{\text{THSR}}_{n\alpha} (X - X_G : B_\perp, B_z) = e^{-\frac{2}{B_\perp^2}(X_x^2 + X_y^2) - \frac{2}{B_z^2}X_z^2}$$

Two limits
- $B = b$: Slater determinant
- $B \gg b$: Gas of independent $\alpha$-particles


Model

- α condensate type wave function (THSR)
- fully microscopic model
- only one parameter, $B$ (with deformation, $B_x, B_y, B_z$) which characterizes nuclear density
- Best description for $^8$Be, Hoyle state ($^{12}$C($0^+_2$)).

$3\alpha$ RGM w.f. (best w.f. ever known)

$$ A\left\{ \chi_{3\alpha}^{RGM}(\xi_1, \xi_2)\phi_{\alpha_1}\phi_{\alpha_2}\phi_{\alpha_3} \right\} = \hat{P}_{g.s.}\hat{P}_{J=0} A\left\{ \prod_{i=1}^{3} \chi_{3\alpha}^{THSR}(X_i : B_{\perp}, B_z)\phi_{\alpha_i} \right\} $$

With $(B_{\perp}, B_z) = (7.6 \text{ fm}, 2.5 \text{ fm})$, one-parameter THSR w.f. gives 99.3 % squared overlap with RGM w.f.

(For $^8$Be, 99.99 %)

$\hat{P}_j$: angular momentum projection operator

$$ \hat{P}_{g.s.} = 1 - |0^+_1\rangle \langle 0^+_1| $$

Y. F. et al., PRC 67, 051306 (2003); 80, 064326 (2009).
Model

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Y. F. et al., PRC 67, 051306 (2003); 80, 064326 (2009).
Hyper-THSR, applied to $^9\Lambda Be, ^{13}\Lambda C, ^{17}\Lambda O, \ldots$

$\Lambda$ particle is a good probe to investigate the analogous states to ordinary nuclei.
- out of antisymmetrization of nucleons
- glue-like role

$$\xi_\Lambda = r_\Lambda - X_C \quad X_C = \frac{r_1 + \cdots + r_{4n}}{4n}$$

$\hat{\mathcal{P}}_i$: angular momentum projection operator

$$\Phi_{\text{Hyper-THSR}}^{[I, l, J]}(B_\perp, B_z, \kappa) = \mathcal{A} \left\{ \prod_{i=1}^{n} \hat{\mathcal{P}}_i \chi_{3\alpha}^{\text{THSR}}(B_\perp, B_z : X_i - X_C) \phi(\alpha_i) \right\} \phi^{(l)}_{\kappa}(\xi_\Lambda)$$

$$\chi^{\text{THSR}}(X : B_\perp, B_z) = \exp \left( -\frac{2}{B_\perp^2} (X_x^2 + X_y^2) - \frac{2}{B_z^2} X_z^2 \right)$$

$$\phi^{(l)}_{\kappa}(\xi_\Lambda) = N_{\kappa,\xi_\Lambda} \exp \left( -\frac{\xi_\Lambda^2}{\kappa^2} \right) Y_{lm}(\xi_\Lambda)$$

In the present study, $l=0$ only taken into account

Validity of this model should be checked.

Application to $^{13}\Lambda C$
Spatial shrinkage is seen.
Comparison with the microscopic model (RGM) calculation ($^{13}_\Lambda^C$)

Progress of Theoretical Physics Supplement No. 81, 1985

Chapter IV. Structure Study of Typical Light Hypernuclei
Taiichi YAMADA, Toshio MOTOB,* Kiyomi IKEDA** and Hiroharu BANDO***

§ 1. Introduction
§ 2. Structure of $^{13}_\Lambda^C$

2.1. Formulation of the microscopic $3\alpha + \Lambda$ model
2.2. Results and discussion
2.3. Summary

$^{12}_N^C : 3\alpha$ RGM wave function ($0_1^+, 2_1^+, 4_1^+, 0_2^+, 2_2^+, 1_1^-, 3_1^-$)

$\Lambda - ^{12}_N^C$ relative part: harmonic oscillator

\[ N = 2n + l : n = 0, 1, 2, 3, 4, l = 0^+, 1^-, 2^+, 3^- \]

\[ V_{NN} : \text{Volkov No. 2 (} M = 0.59 \text{)} \]

\[ V_{AN} = \nu_{AN}^0 \exp \left( -\frac{(r_N - r_\Lambda)^2}{\beta_{AN}^2} \right), \quad \nu_{AN}^0 = -35.4 \text{ [MeV]}, \quad \beta_{AN} = 1.034 \text{ [fm]} \]
<table>
<thead>
<tr>
<th>$L^*$</th>
<th>Main channel</th>
<th>Full (MeV)</th>
<th>NO (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_{11}$</td>
<td>$[^{12}\text{C}(0^+_{11}) \otimes s_4]$</td>
<td>$-11.31$</td>
<td>$-10.11$</td>
</tr>
<tr>
<td>$2^+_{11}$</td>
<td>$[^{12}\text{C}(2^+_{11}) \otimes s_4]$</td>
<td>$-6.87$</td>
<td>$-6.12$</td>
</tr>
<tr>
<td>$4^+_{11}$</td>
<td>$[^{12}\text{C}(4^+_{11}) \otimes s_4]$</td>
<td>$1.73$</td>
<td>$2.11$</td>
</tr>
<tr>
<td>$0^+_{10}$</td>
<td>$[^{12}\text{C}(0^+_{10}) \otimes s_4]$</td>
<td>$3.36$</td>
<td>$4.85$</td>
</tr>
<tr>
<td>$2^+_{10}$</td>
<td>$[^{12}\text{C}(2^+_{10}) \otimes s_4]$</td>
<td>$8.04$</td>
<td>$8.80$</td>
</tr>
<tr>
<td>$3^+_{10}$</td>
<td>$[^{12}\text{C}(3^-_{10}) \otimes s_4]$</td>
<td>$2.76$</td>
<td>$3.35$</td>
</tr>
<tr>
<td>$1^-_{10}$</td>
<td>$[^{12}\text{C}(1^-_{10}) \otimes s_4]$</td>
<td>$6.43$</td>
<td>$7.36$</td>
</tr>
</tbody>
</table>
Energy curve of $^{13}_\Lambda$C($0^+$) as a function of $\kappa$

$$\sum_{B'_1, B'_2} \langle \Phi_{[0,0],0}^{\text{Hyper-THSR}}(B'_1, B'_2, \kappa) | H - E_{\lambda}(\kappa) | \Phi_{[0,0],0}^{\text{Hyper-THSR}}(B'_1, B'_2, \kappa) \rangle f_{\lambda}(B'_1, B'_2) = 0$$

$\Lambda$ particle w.f. $\phi_{\kappa}^{(l=0)}(\xi_\Lambda) = N_{\kappa,l=0} \exp\left(-\frac{\xi_\Lambda^2}{K^2}\right) Y_{00}(\xi_\Lambda)$

3 minima appear
Energy of $^{13}_\Lambda$C(0$^+$)

- Hyper-THSR gives better results than RGM+Λ.

\[
\sum_{B'_1, B'_2, \kappa'} \left\langle \Phi_{[0,0]}^{\text{Hyper-THSR}}(B'_1, B'_2, \kappa') \right| H - E_\lambda \left| \Phi_{[0,0]}^{\text{Hyper-THSR}}(B'_1, B'_2, \kappa') \right\rangle f_\lambda(B'_1, B'_2, \kappa') = 0
\]

-101.8
-100.7
-101.2
-86.04
-85.9
-81.8
-83.5
-87.3

RGM+Λ
Hyper-THSR(min.)
Hyper-THSR

RGM+Λ: Yamada et al., PTPS 81, (1985)
Energy of $^{13}_\Lambda$C(0\(^+\))

- Hyper-THSR gives better results than RGM+\(\Lambda\).
- \(0_3^+\) state is clearly obtained. (failed in \(^{12}\)C)
  This must be deeply related to \(0_3^+\) in \(^{12}\)C

Further analyses are now going on.

RGM+\(\Lambda\): Yamada et al., PTPS 81, (1985)
Nucleon density of the core \(^{12}\text{C}\)

Density operator defined by

\[
\rho(r) = \frac{1}{12} \sum_{i=1}^{12} \delta(|r_i - X_c| - r)
\]

\(R_{\text{rms}}\) : rms radius of the core \(^{12}\text{C}\)

\[X_c = \frac{r_i + \cdots + r_{12}}{12}\]
Nucleon density of the core ($^{12}\text{C}$)

Smaller effect of shrinkage for the g.s.

Density operator defined by

$$\rho(r) = \frac{1}{12} \sum_{i=1}^{12} \delta(|r_i - X_c| - r)$$

$$X_c = \frac{r_1 + \cdots + r_{12}}{12}$$

$R_{\text{rms}}$: rms radius of the core ($^{12}\text{C}$)

$^{12}\text{C}(0^+_1)$

$^{12}\text{C}(0^+_2)$

$^{13}\text{C}(0^+_1)$
Nucleon density of the core \(^{12}\text{C}\)

Smaller effect of shrinkage for the g.s. than \(^{8}\text{Be}\)

\[ R_{\text{rms}} = 2.4 \text{ fm} \rightarrow 2.2 \text{ fm} \]

\[ R_{\text{rms}} = 3.8 \text{ fm} \]

Density operator defined by

For \(^{8}\text{Be}\)

\[ 8\text{Be}(0^+_1) \]

\[ \Lambda_{^{9}\text{Be}}(0^+_1) \]

\[ 12\text{C}(0^+_1) \]

\[ 12\text{C}(0^+_2) \]

\[ \Lambda_{^{13}\text{C}}(0^+_1) \]

\[ X_c = \frac{1}{12} \]

\( R_{\text{rms}} \): rms radius of the core \((^{12}\text{C})\)
Nucleon density of the core ($^{12}$C)

Very large effect of shrinkage for the Hoyle state

Density operator defined by

$$\rho(r) = \frac{1}{12} \sum_{i=1}^{12} \delta(|r_i - X_c| - r)$$

$R_{\text{rms}}$ : rms radius of the core ($^{12}$C)

For example:

$R_{\text{rms}} = 2.4 \text{ fm} \rightarrow 2.2 \text{ fm}$

$R_{\text{rms}} = 3.8 \text{ fm} \rightarrow 2.8 \text{ fm}$
Nucleon density of the core ($^{12}$C)

Very large effect of shrinkage for the Hoyle state

Density operator defined by

$$\rho(r) = \frac{1}{12} \sum_{i=1}^{12} \delta(|r_i - X_c| - r)$$

$$R_{\text{rms}} : \text{rms radius of the core} \ (^{12}\text{C})$$

$$X_c = \frac{r_1 + \cdots + r_{12}}{12}$$

$R_{\text{rms}} = 2.4 \text{ fm} \rightarrow 2.2 \text{ fm}$

$R_{\text{rms}} = 3.8 \text{ fm} \rightarrow 2.8 \text{ fm}$

$R_{\text{rms}} = 3.1 \text{ fm}$
Size dependence of occupation probability

(amount of $\alpha$ condensation)

Condensate character still much survives.

$\sim 60\%$

$R_{\text{rms}} < 2.5$ fm: Alpha’s are resolved due to the antisymmetrization.

$R_{\text{rms}} \rightarrow$ large: Alpha’s occupy a single $S$-orbit only.
summary

- Fully microscopic alpha-condensate type w.f. (Hyper-THSR)
  - giving better description of $^{13}\Lambda C$ than (CC-)RGM
  - core-rearrangement effect is naturally and effectively taken into account

- $^{0,2}_2^+ : \alpha$ condensate character survives.

- $^{0,3}_3^+ :$ large components of $\alpha$ condensation
  Relation with $^{12}C(O_3^+)$ ?

- YNG interaction (ESC08a,b,“a”)
- Complex Scaling Method (elimination of $\Lambda +^{12}C$ continuums)