Updated study of S-wave $\bar{K}N$ scattering and related spectroscopy at NLO in UChPT

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1. Introduction. Interest.
2. S-Wave, $S=-1$ Meson-Baryon Scattering
3. Scattering
4. Spectroscopy
5. Conclusions
1. INTRODUCTION. INTEREST.

$\bar{K}N$ scattering: Ten two-body coupled channels

$\pi^0\Lambda(1.25)\pi^0\Sigma^0\pi^-\Sigma^+\pi^+\Sigma^-(1.33)K^-p\bar{K}^0p(1.43)$
$\eta\Lambda(1.66)\eta\Sigma^0(1.74)K^0\Xi^0K^-\Xi^+(1.81)$

Explicit chiral symmetry breaking: Large SU(3) breaking effects due to the large splittings between the thresholds of the ten coupled channels

Important isospin breaking effects due to cusps at thresholds, we work with the physical basis.

Precise measurement of energy shift and width of kaonic hydrogen $1s$ state: SIDDHARTA Collaboration (2011)

Achieve a precise extrapolation of the $\bar{K}N$ scattering amplitude below threshold

This is important for studies of possible $\bar{K}$-nuclear clusters
Renewed interest with the precise measurement by SIDDHARTA Coll. of strong shift and width of kaonic hydrogen $1s$ energy level

M. Bazzi et al., PLB704,113(’11)

Upwards shift
Repulsive $K^-p \rightarrow K^-p$

$$K^-p \rightarrow \begin{cases} 
\pi^0 \Lambda, \pi^\mp \Sigma^\pm \text{[strong]} \\
\Sigma \pi \gamma, \Sigma \pi e^+ e^-, \Sigma \gamma, \ldots < 1\% 
\end{cases}$$

Unstable

**DEAR:**

$\Delta E = 193 \pm 37\,(\text{stat.}) \pm 6\,(\text{syst.})$ eV

$\Gamma = 249 \pm 111\,(\text{stat.}) \pm 39\,(\text{syst.})$ eV

**KEK:**

$\Delta E = 323 \pm 63 \pm 11$ eV

$\Gamma = 407 \pm 208 \pm 100$ eV.

Meissner,Raha,Rusetsky EPJ C35,349(’04); Borasooy,Nissler,Weise PRL94,213401(05), EPJA25,79(05) pointed out a possible inconsistency between DEAR and previous scattering data. However, it was still possible to obtain compatible fits Prades,Verbeni,JAO PRL95,172502 (‘05); JAO, EPJA28,63(‘06)
\[
E_{1s} = E_{1s}^{em} + \epsilon_{1s}, \quad \epsilon_{1s} \text{ is complex}
\]

Deser Formula \[
\epsilon_{1s} = -2\alpha^3 \mu_C^2 T_{K^-p}
\]

Precise knowledge \[\epsilon_{1s} \leftrightarrow T_{K^-p} \text{ at threshold}\]

Meissner, Raha, Rusetsky EPJ C35, 349 (’04) include isospin breaking correction on the Deser formula up to an including \(O(\alpha^4, \alpha^3(m_u-m_d)) \sim 9\%\)

Cusp Effect: \(\sim 50\% \ O(\delta^{1/2})\)

Coulomb Effects: \(\sim 10 - 15\%\)

Vacuum Polarization: \(\sim 1\%\)

\[
\Delta E_{1s} - \frac{1}{2} \Gamma_{1s} = -\frac{\alpha^3 \mu_C^3}{2\pi M_{K^+}} T_{K^-p} \left\{ 1 - \frac{\alpha \mu_C^2 s_1(\alpha)}{4\pi M_{K^+}} T_{K^-p} \right\}
\]

\(\delta \sim \alpha \sim m_u - m_d\)

DEAR/SIDDHARTA Coll.’s aim was to finally measure it up to the eV level, a few percent (nowadays the precision is 20\%)

\[
\Delta E = 283 \pm 36 \text{ eV}, \quad \Gamma = 541 \pm 92 \text{ eV}
\]
Other interesting topics for which a precise knowledge of the $\bar{K}N$ scattering amplitude is important:

- **Nature of $\Lambda(1405)$, problems in lattice QCD. Dynamically generated resonance.**
  
  Meissner, J.A.O, PLB500, 263(’01); Jido, Oset, Ramos, Meissner, J.A.O, NPA725(03)181

- **Two poles making up the $\Lambda(1405)$**
  
  Magas, Oset, Ramos, PRL95, 052301(’05); S. Prakhov et al. (Crystall Ball Coll.), PRC70, 034605(’04);

- **Strangeness content of the proton and large pion-nucleon sigma terms,**
  
  $\langle p|\bar{s}s|p\rangle$ strange proton-scalar form factor related by unitarity with $\bar{K}N$ amplitudes.

- **Strangeness photoproduction**
- Potential Models, Quark Models, (Chiral) Bag Models, etc
- CHPT+Unitarization (UChPT)

Kaiser, Siegel, Weise NPA594, 325 ('95)
Oset, Ramos NPA635, 99 ('98)
Meissner, JAO PLB500, 263 ('01)
Lutz, Kolomeitsev NPA700, 193 ('02);
Garcia-Recio, Lutz, Nieves, PLB582, 49 ('04);
Borasoy, Nissler, Weise PRL94, 213401 (05), EPJA25, 79 ('05)
Borasoy, Meissner, Nissler, PRC74, 055201 ('06)

Most recent studies prompted by the SIDDHARTA measurement
Ikeda, Hyodo, Weise PLB706, 63 (2011); NPA881, 98 (2012)
2. Formalism

$\mathcal{O}(p)$ and $\mathcal{O}(p^2)$ SU(3) Chiral Lagrangians

\[ \mathcal{L}_1 = \langle i \bar{B} \gamma^\mu [D_\mu, B] \rangle - m_0 \langle \bar{B} B \rangle + \frac{D}{2} \langle \bar{B} \gamma^\mu \gamma_5 \{u_\mu, B\} \rangle + \frac{F}{2} \langle \bar{B} \gamma^\mu \gamma_5 [u_\mu, B] \rangle, \]

$D = 0.8, F = 0.46, m_0 =$ baryon mass in SU(3) chiral limit

\[ \mathcal{L}_2 = b_0 \langle \bar{B} B \rangle \langle \chi_+ \rangle + b_D \langle \bar{B} \{\chi_+, B\} \rangle + b_F \langle \bar{B} [\chi_+, B] \rangle + b_1 \langle \bar{B} [u_\mu, [u^\mu, B]] \rangle + b_2 \langle \bar{B} \{u_\mu, \{u^\mu, B\}\} \rangle + b_3 \langle \bar{B} \{u_\mu, [u^\mu, B]\} \rangle + b_4 \langle \bar{B} B \rangle \langle u_\mu u^\mu \rangle + \ldots \]

$U = e^{i \Phi/f}, U = u^2, u = e^{i \Phi/2f}, u_\mu = iu^\dagger (\partial_\mu U) u^\dagger$

$\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u, \chi = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$

$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ -\frac{\pi^-}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^- & \bar{K}^0 \\ \frac{2}{\sqrt{6}} \eta & -2 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix}$

$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ -\frac{\Sigma^-}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Xi^- & \Xi^0 \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}$
• Enhancement of the unitarity cut that makes definitively smaller the overall scale $\Lambda_{\text{CHPT}}$ in meson-baryon scattering with strangeness:

Arbitrary Meson-Baryon Vertex

The presence of large masses compared with the typical low three-momenta (Baryon+Kaon masses) drives the appearance of the $\Lambda(1405)$ close to threshold in $\overline{K}N$ scattering.

This also occurs e.g. in Nucleon-Nucleon scattering due to the nucleon mass
Let us keep track of the kaon mass, $M_K \approx 500\text{MeV}$.

We follow similar arguments to those of S. Weinberg in *NPB363,3* (’91) for NN scattering (nucleon mass).

**Unitarity Diagram**

$$\int \frac{dq^0}{(k^0 - q^0 + i\epsilon)(q^0 + E(q) - i\epsilon)(q^0 - E(q) + i\epsilon)}$$

**Unitarity enhancement for low three-momenta:**

$$\frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \approx \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K}$$

Around one order of magnitude in the region of the $\Lambda(1405)$ region, $|q| \approx 100\text{MeV}$.
\[ M_K \approx 500 \]

Let us take now the crossed diagram
\[ k \rightarrow -k \]
\[ \frac{1}{k^0 - E(q)} \frac{1}{2E(q)} \approx \frac{2M_K}{k^2 - q^2} \frac{1}{2M_K} \]

Unitarity&Crossed loop diagram:
\[ \frac{4M_K^2}{k^2 - q^2} \]

Unitarity enhancement for low three-momenta:
\[ \frac{2M_K}{q} \]
The unitarity cut (sum over the unitarity bubbles) is enhanced

\[ UChPT \text{ makes an expansion of the ``Interacting Kernel'' from } SU(3) \text{ baryon ChPT. In our study up to } O(p^2) \]
General Expression for a Partial Wave Amplitude

- Above threshold and on the real axis (physical region), a partial wave amplitude must fulfill because of unitarity:

\[ \text{Im} \ T_{ij} = \sum_k T_{ik} \rho_k T_{kj}^* \rightarrow \text{Im} \ T_{ij}^{-1} = -\rho_i \delta_{ij} \]  

Unitarity Cut

We perform a dispersion relation for the inverse of the partial wave (the discontinuity when crossing the unitarity cut is known):

\[ T_{ij}^{-1} = R_{ij}^{-1} + \delta_{ij} \left( g(s_0)_i - \frac{s - s_0}{\pi} \int_{s_{th,i}}^{\infty} \frac{\rho(s')_i}{(s' - s - i0^+)(s' - s_0)} ds' \right) \]

The rest

\[ g(s)_i: \text{ Single unitarity bubble} \]
1. \( T \) is calculated in SU(3) baryon ChPT \( T = T_1 + T_2 + T_3 + \ldots \).

2. \( R \) is fixed by matching algebraically expressions of \( T \),

\[
R = R_1 + R_2 + R_3 + \ldots
\]

\[
T_1 + T_2 + T_3 = R_1 + R_2 - R_1^2 g
\]

\[
R_1 = T_1
\]

\[
R_2 = T_2
\]

3. Crossed channel dynamics is included perturbatively in the ChPT calculation of \( T \).
3. S-WAVE, S=-1 MESON-BARYON SCATTERING

Z.-H. Guo, J.A.O., forthcoming

\[ T = \left[ R^{-1} + g(s) \right]^{-1} = \left[ I + R \cdot g(s) \right]^{-1} \cdot R(s) \]

\[ R = R_1 = T_1 \quad \text{LEADING ORDER, } \mathcal{O}(p) \]

\[ R = R_1 + R_2 = T_1 + T_2 \quad \text{NLO, } \mathcal{O}(p^2) \]

for \( \mathcal{O}(p^3) \) and higher \( R_n \neq T_n \)

\( \mathcal{O}(p) \) from \( \mathcal{L}_1 \)

\( \mathcal{O}(p^2) \) from \( \mathcal{L}_2 \)
S-wave projection of ChPT amplitudes

\[ R_{ij} = \frac{1}{4\pi} \int d\Omega \, V_{ij}(W, \Omega, \sigma_i, \sigma_i) \]

Cross sections

\[ \sigma(M_iB_i \rightarrow M_jB_j) = \frac{1}{16\pi s} \frac{|\vec{p}_j|}{|\vec{p}_i|} |T_{M_iB_i \rightarrow M_jB_i}|^2 \]
I) DATA INCLUDED IN THE FITS

1) CROSS SECTIONS:

\[ K^- p \rightarrow K^- p , \bar{K}^0 n , \pi^+ \Sigma^- , \pi^- \Sigma^+ , \pi^0 \Sigma^0 , \pi^0 \Lambda \]

In the fit we include data from threshold up to \( p_{lab} = 0.3 \) GeV.

2) Precisely Measured Ratios 5%

\[ \gamma = \frac{\sigma(K^- p \rightarrow \pi^+ \Sigma^-)}{\sigma(K^- p \rightarrow \pi^- \Sigma^+)} = 2.36 \pm 0.12 \]

\[ R_c = \frac{\sigma(K^- p \rightarrow \text{charged particles})}{\sigma(K^- p \rightarrow \text{all})} = 0.664 \pm 0.033 \]

\[ R_n = \frac{\sigma(K^- p \rightarrow \pi^0 \Lambda)}{\sigma(K^- p \rightarrow \text{all neutral states})} = 0.189 \pm 0.015 \]
3) $\pi \Sigma$ EVENT DISTRIBUTION AROUND THE $\Lambda(1405)$ RESONANCE

4) SIDDHARTA STRONG SHIFT AND WIDTH OF KAONIC HYDROGEN

5) WE ALSO CONSTRAINT OUR FITS CALCULATING AT $O(p^2)$ IN PURE BARYON CHPT SEVERAL PION-NUCLEON OBSERVABLES, WHERE CHPT EXPANSION IS RELIABLE:

$$\sigma_{\pi N} = -2m_{\pi}^2 (2b_0 + b_F + b_D), \quad b_i \text{ from the fits}$$

$$a_{0+}^+ = \frac{m_{\pi}^2}{2\pi f^2} \left(-2b_1 + b_2 + b_3 - \frac{g_A^2}{8m}\right)$$

$\sigma_{\pi N} = 30 \pm 20 \text{ MeV}$ (45 $\pm$ 8 MeV from Gasser, Leutwyler, Sainio PLB253, 252 (’91), 59 $\pm$ 7 MeV Alarcón, Martin-Camalich, JAO PRD85, 051503(R) (‘12); higher order corrections $\pm 10$ MeV Gasser, AP254, 192 (’97))

$a_{0+}^+ = (0 \pm 1) m_{\pi}^{-1} 10^{-2}$ Baru et al., PLB694, 473 (‘11) (7.6 $\pm$ 3.1) $m_{\pi}^{-1} 10^{-3}$ and expected higher order corrections $+m_{\pi} 10^{-2}$ from unitarity Bernard et al. PLB309, 421 (’93).

We also include in the fit the baryon masses calculated at $O(p^2)$ in ChPT: $m_N, m_\Lambda, m_\Sigma, m_\Xi$ 30% error
I) RECENT FURTHER DATA INCLUDED IN THE EXTENDED ANALYSIS  *JAO EPJA*28,63(2006), not in the most recent studies of Ikeda, Hyodo, Weise nor of Mai, Meissner

6) $\sigma(K^- p \rightarrow \eta \Lambda)$ cross-section

On top of the $\Lambda(1670)$ resonance.

7) $\sigma(K^- p \rightarrow \Sigma^0 \pi^0 \pi^0)$

total cross-section and event distribution.

6) and 7) measured by the Crystall-Barrell Collaboration, 2001 and 2004, respectively. Precise experimental data.

8) $\Lambda \pi$ P- and S-wave phase shift difference at $\Xi^-$ mass $\delta_P - \delta_S = (3.2 \pm 5.3)^\circ$.

E756 Coll. *PRL*91,031601 (’03)
For the calculation of the process $K^- p \rightarrow \pi^0 \pi^0 \Sigma^0$ we take as the production vertex the mechanism:

Which dominates due to the almost on-shell character of the intermediate proton.

The solid point means full $K^- p \rightarrow \pi^0 \Sigma^0$ S-wave

Magas, Oset, Ramos
PRL95,052301(’05).
JAO EPJA28,63(2006).
Two sources of uncertainty, overlooked in other studies, are discussed

1.- Use of a **FIT I**: common $f$ (pseudoscalar weak decay constant) or **FIT II**: distinguishing between $f_\pi$, $f_K$, $f_\eta$

It gives rise to a rather large uncertainty in the subthreshold extrapolation of the $K$-$p$ scattering amplitude

2.- Two $\chi^2$ definitions used in the literature:

**Weighted $\chi^2$ per observable:**

$$\chi_{d.o.f}^2 = \frac{\sum_k n_k}{K(\sum_k n_k - n_p)} \sum_{k=1}^K \chi_k^2$$ (1)

**Common $\chi^2$ definition:**

$$\chi_{d.o.f}^2 = \frac{1}{\sum_k n_k - n_p} \sum_{k=1}^K \chi_k^2$$ (2)

Our fits are quite stable under the change of the $\chi^2$ definition
All the subtraction constants $a_i$ have natural size $O(1)$

The same applies to the $b_i$

### Fits I and II: Good reproduction of data with $\chi^2_{d.o.f}$ smaller than 1

### $O(p)$ Fit: It also gives rise to quite a good fit. It fails to reproduce $\sigma(K^-p \rightarrow \eta\Lambda)$. Removing these data $\chi^2_{d.o.f} \simeq 1.23$
FIT I

Solid: $\chi^2$ weighted per observable

Dashed: $\chi^2$ common definition
FIT II

Solid: $\chi^2$ weighted per observable

Dashed: $\chi^2$ common definition
$O(p)$-Fit

**Solid:** $\chi^2$ weighted per observable

**Dashed:** $\chi^2$ common definition

*Problem*
Interpret:

**Systematic uncertainty**: Treatment of $f$’s, definition for $\chi^2$, higher orders, …

**Spread of central values for fits I and II**

We calculate the mean and variance

**Statistical uncertainty**: largest error bars from common $\chi^2$ def.
Interpret:

**Systematic uncertainty:** Treatment of $f$’s, definition for $\chi^2$, higher orders,…

Spread of central values for fits I and II

We calculate the mean and variance

**Statistical uncertainty:** largest error bars from common $\chi^2$ def.
K⁻ P SCATTERING LENGTH:

Martin, NPB179,33(‘81): ................................................................. \( a_{K-p} = -0.67 + i 0.64 \text{ fm} \)

Kaiser,Siegel,Weise, NPA594,325(‘95): .......................... \( a_{K-p} = -0.97 + i 1.1 \text{ fm} \)

Oset,Ramos, NPA635,99(‘98): .......................................................... \( a_{K-p} = -0.99 + i 0.97 \text{ fm} \)

Meissner,JAO PLB500,263(‘01): .................................................. \( a_{K-p} = -0.75 + i 1.2 \text{ fm} \)

Borasoy,Nissler,Weise, PRL94,213401(‘05), EPJA25,79(‘05): ... \( a_{K-p} = -0.51 + i 0.82 \text{ fm} \)

Prades,Verbeni,JAO PRL95,172502(‘05): ................................. \( A_4^+: a_{K-p} = -0.51 + i 0.42 \text{ fm} \)
\( B_4^+: -1.01 + i 0.80 \text{ fm} \)

JAO, EPJA28,63(‘06) : ................................................................. \( A\text{-fit}: a_{K-p} = -0.5 + i 0.41 \text{ fm} \)
\( B\text{-fit}: a_{K-p} = -1.0 + i 1.0 \text{ fm} \)

Ikeda, Hyodo, Weise NPA881,98(2012).................................. \( a_{K-p} = -0.70 + i 0.89 \text{ fm} \)
\( a_{K-n} = 0.57^{+0.04}_{-0.21} + i 0.72^{+0.26}_{-0.41} \text{ fm} \)

THIS WORK: (PRELIMINARY)

\[ a_{K^-p} = (-0.70 \pm 0.10) + i (0.94 \pm 0.10) \text{ fm} \]
\[ a_{K^-n} = (+0.36 \pm 0.07) + i (0.56 \pm 0.08) \text{ fm} \]
K - P SUBTHRESHOLD EXTRAPOLATION:

The uncertainty due to the change of fit is much larger than the statistical error band. An $O(p^3)$ calculation is called for.
5. POLE CONTENT

\[ T_{ij} = - \lim_{s \to s_R} \frac{\gamma_i \gamma_j}{s - s_R} \]

Residues

Pole Position \( \approx (M_R - i \Gamma_R/2)^2 \)

Physical Riemann Sheet

<table>
<thead>
<tr>
<th>( \pi \Lambda )</th>
<th>( \pi \Sigma )</th>
<th>( \bar{K}N )</th>
<th>( \eta \Lambda )</th>
<th>( \eta \Sigma )</th>
<th>( K \Xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25 1.33 1.43</td>
<td></td>
<td></td>
<td>1.66 1.74 1.81</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Different Riemann Sheets:

2RS 3RS 4RS 5RS 6RS 7RS

GeV
Two pole structure of $\Lambda(1405)$ re-confirmed

Heavier and narrower pole coupling more strongly to $K\bar{N}$

Lighter and broader pole coupling strongly to $\pi\Sigma$

We also obtain the $\Lambda(1670)$ in good agreement with properties in PDG

| Pole               | $|\beta_{\pi\Lambda}|$ | $|\beta_{\pi\Sigma}|_0$ | $|\beta_{\pi\Sigma}|_1$ | $|\beta_{\pi\Sigma}|_2$ | $|\beta_{K\bar{N}}|_0$ | $|\beta_{K\bar{N}}|_1$ | $|\beta_{\eta\Lambda}|$ | $|\beta_{\eta\Sigma}|$ | $|\beta_{K\Xi}|_0$ | $|\beta_{K\Xi}|_1$ |
|--------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $\Lambda(1405)$    |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |
| $1436^{+11}_{9} - i 126^{+17}_{-23}$ (3RS) | 0.0^{+0.0}_{-0.0} | 8.8^{+0.6}_{-0.6} | 0.0^{+0.0}_{-0.0} | 0.0^{+0.0}_{-0.0} | 7.7^{+0.9}_{-0.7} | 0.0^{+0.0}_{-0.0} | 1.4^{+0.3}_{-0.2} | 1.4^{+0.4}_{-0.2} | 2.1^{+0.8}_{-0.3} | 1.5^{+0.5}_{-0.4} | 0.0^{+0.0}_{-0.0} |
| $1417^{+2}_{-4} - i 24^{+5}_{-3}$ (3RS)   | 0.1^{+0.0}_{-0.0} | 5.0^{+1.0}_{-1.0} | 0.1^{+0.0}_{-0.0} | 0.0^{+0.0}_{-0.0} | 7.7^{+0.8}_{-0.4} | 0.1^{+0.0}_{-0.0} | 1.4^{+0.4}_{-0.2} | 0.1^{+0.0}_{-0.0} | 1.5^{+0.5}_{-0.4} | 0.0^{+0.0}_{-0.0} |
| $\Lambda(1670)$    |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |
| $1674^{+3}_{-2} - i 8^{+3}_{-2}$ (4RS)    | 0.0^{+0.0}_{-0.0} | 0.8^{+0.2}_{-0.1} | 0.0^{+0.0}_{-0.0} | 0.0^{+0.0}_{-0.0} | 1.5^{+0.2}_{-0.2} | 0.0^{+0.0}_{-0.0} | 1.5^{+0.2}_{-0.1} | 1.5^{+0.2}_{-0.1} | 10.8^{+0.2}_{-0.2} | 0.1^{+0.0}_{-0.0} |
| $1674^{+3}_{-2} - i 11^{+3}_{-2}$ (5RS)   | 0.0^{+0.0}_{-0.0} | 0.9^{+0.2}_{-0.2} | 0.0^{+0.0}_{-0.0} | 0.0^{+0.0}_{-0.0} | 1.6^{+0.2}_{-0.2} | 0.0^{+0.0}_{-0.0} | 1.7^{+0.3}_{-0.2} | 0.0^{+0.0}_{-0.0} | 11.1^{+0.3}_{-0.2} | 0.1^{+0.0}_{-0.0} |
| $1673^{+3}_{-2} - i 11^{+3}_{-2}$ (6RS)   | 0.0^{+0.0}_{-0.0} | 0.9^{+0.2}_{-0.1} | 0.0^{+0.0}_{-0.0} | 0.0^{+0.0}_{-0.0} | 1.6^{+0.2}_{-0.2} | 0.0^{+0.0}_{-0.0} | 1.7^{+0.3}_{-0.2} | 0.0^{+0.0}_{-0.0} | 11.1^{+0.3}_{-0.2} | 0.1^{+0.0}_{-0.0} |
| $\Sigma I = 1$    |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |
| $1646^{+21}_{-55} - i 160^{+58}_{-30}$ (4RS) | 3.1^{+1.3}_{-0.3} | 0.0^{+0.0}_{-0.0} | 3.0^{+0.1}_{-0.0} | 0.0^{+0.0}_{-0.0} | 2.9^{+0.3}_{-0.2} | 0.0^{+0.0}_{-0.0} | 7.9^{+0.6}_{-0.4} | 0.0^{+0.0}_{-0.0} | 6.4^{+0.9}_{-1.2} | 16.1^{+2.4}_{-1.6} |
| $1878^{+45}_{-39} - i 169^{+21}_{-34}$ (6RS) | 1.0^{+0.2}_{-0.4} | 0.0^{+0.0}_{-0.0} | 5.8^{+0.9}_{-0.6} | 0.0^{+0.0}_{-0.0} | 3.7^{+0.3}_{-0.3} | 0.0^{+0.0}_{-0.0} | 3.9^{+1.1}_{-0.0} | 0.1^{+0.0}_{-0.0} | 6.4^{+0.9}_{-1.2} | 16.1^{+2.4}_{-1.6} |
**FIT II**

| Pole                      | $|\beta_{\pi\Lambda}|$ | $|\beta_{\pi\Sigma}|_0$ | $|\beta_{\pi\Sigma}|_1$ | $|\beta_{\pi\Sigma}|_2$ | $|\beta_{K\Lambda}|_0$ | $|\beta_{K\Lambda}|_1$ | $|\beta_{\eta\Lambda}|$ | $|\beta_{\eta\Sigma}|$ | $|\beta_{K\Xi}|_0$ | $|\beta_{K\Xi}|_1$ |
|---------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $1376^{+2}_{-2} - i 33^{+5}_{-4}$ (3RS) | 2.0$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 0.1$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 0.1$^{+0.0}_{-0.0}$ | 2.1$^{+0.4}_{-0.3}$ | 0.0$^{+0.0}_{-0.0}$ | 4.0$^{+0.3}_{-0.2}$ | 0.0$^{+0.0}_{-0.0}$ | 6.3$^{+0.2}_{-0.1}$ |
| $1414^{+3}_{-3} - i 12^{+4}_{-3}$ (3RS) | 1.9$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 1.0$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 0.4$^{+0.2}_{-0.1}$ | 2.5$^{+0.3}_{-0.2}$ | 0.0$^{+0.0}_{-0.0}$ | 3.3$^{+0.3}_{-0.2}$ | 0.1$^{+0.0}_{-0.0}$ | 3.3$^{+0.3}_{-0.2}$ |
| $1686^{+18}_{-18} - i 101^{+8}_{-7}$ (5RS) | 0.2$^{+0.1}_{-0.0}$ | 0.0$^{+0.0}_{-0.0}$ | 3.5$^{+0.2}_{-0.2}$ | 0.0$^{+0.0}_{-0.0}$ | 0.0$^{+0.0}_{-0.0}$ | 3.5$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 3.9$^{+0.3}_{-0.3}$ | 0.1$^{+0.0}_{-0.0}$ | 10.9$^{+0.2}_{-0.2}$ |
| $1741^{+10}_{-12} - i 94^{+2}_{-3}$ (6RS) | 1.1$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 2.3$^{+0.2}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 0.0$^{+0.0}_{-0.0}$ | 2.8$^{+0.1}_{-0.1}$ | 0.0$^{+0.0}_{-0.0}$ | 3.7$^{+0.2}_{-0.2}$ | 0.1$^{+0.0}_{-0.0}$ | 7.9$^{+0.2}_{-0.2}$ |

**Similar $I=1$ poles around the KbarN threshold were reported in Meissner, JAO PLB500,263('01); Prades, Verbeni, JAO PRL95,172502('05); JAO EPJA28,63('06)**
INFLUENCE OF THE I=1 RESONANCES IN $\pi\Sigma$ EVENT DISTRIBUTION

$\gamma p \rightarrow K^+\Lambda(1405)$
$\rightarrow K^+\pi^+\Sigma^-, K^+\pi^-\Sigma^+$

J.K. Ahn, NP A721 (’03) 715c

$K^-p \rightarrow \Sigma^{\pm}\pi^{\mp}\pi^-$

Hemingway, NPB253, 742(’85)

LINE:
Nacher, Oset, Toki, Ramos PL B455 (’99)55
4. CONCLUSIONS

1. A UCHPT study of meson-baryon dynamics with strangeness $=-1$ in S-wave up to NNLO or $O(p^2)$

2. We reproduce simultaneously scattering data, including the recent and precise results from the Crystall Ball Collaboration, and atomic data on kaonic hydrogen given by the SIDDHARTA Collaboration.

3. Scattering data and kaonic hydrogen measurement by SIDDHARTA are consistent.

4. We study two sources of ambiguity:
   1) Use of a common pseudoscalar weak decay constant or distinguishing between $f_\pi, f_K$ and $f_\eta$.
   2) Two definitions of $\chi^2$

5. 4.1) increases significantly the uncertainty in the subthreshold extrapolation of the $K^-p$ scattering amplitude
6) To improve the knowledge of the subthreshold extrapolation of the K-p amplitude requires an $O(p^3)$ calculation. Then our kernel will be sensitive to the change in the f’s.

7) We re-confirm the two pole structure of the $\Lambda(1405)$. We also reproduce the $\Lambda(1670)$.

In Fit II we also have some strength around the KbarN threshold in I=1.

8) Scattering lengths

\[ a_{K^-p} = (-0.70 \pm 0.10) + i (0.94 \pm 0.10) \text{ fm} \]

\[ a_{K^-n} = (+0.36 \pm 0.06) + i (0.56 \pm 0.08) \text{ fm} \]
\( K^- p \rightarrow \Sigma^{\pm} \pi^\mp \pi^- \pi^+ \) from them \( \Sigma^{\pm} \pi^\mp \) event distributions are obtained. The \( I=0 \) corresponds to the average of both.

Typically one takes:

\[
\frac{dN_{\pi\Sigma}}{dE} = C \left| T_{\pi\Sigma \rightarrow \pi\Sigma}^{I=0} \right|^2 p_{\pi\Sigma}
\]

As if the process were elastic

E.g: Dalitz, Deloff, JPG 17, 289 (‘91); Müller, Holinde, Speth NPA 513, 557 (‘90), Kaiser, Siegel, Weise NPB 594, 325 (‘95); Oset, Ramos NPA 635, 99 (‘89)

But the \( \bar{K}N \) threshold is only 100 MeV above the \( \pi\Sigma \) one, comparable with the widths of the present resonances in this region and with the width of the shown invariant mass distribution. This prescription is ambiguous, why not?

\[
\frac{dN_{\pi\Sigma}}{dE} = C \left| T_{\bar{K}N \rightarrow \pi\Sigma}^{I=0} \right|^2 p_{\pi\Sigma}
\]

We follow the Production Process scheme previously shown: already employed for this case in Meissner, JAO PLB 500, 263 (‘01)

\[
F = (I + R \cdot g)^{-1} \cdot \xi, \quad \xi^T = (0, r_1, r_1, r_1, r_2, r_2, 0, 0, 0, 0)
\]

\[
\frac{r_2}{r_1} = -0.28
\]

I=0 Source

\( r_2=0 \) (previous approach)
a) is more than twice wider than b) (Quite Different Shape)

b) Couples stronger to $\bar{K}\Sigma$ than to $\pi\Sigma$ contrarily to a)

It depends to which resonance the production mechanism couples stronger that the shape will move from one to the other resonance.
The set of Feynman diagrams contributing to the energy shift of the kaonic hydrogen up-to-and-including \( \mathcal{O} (\alpha^4, \alpha^3(m_d - m_u)) \). Solid, dashed, double, dotted, wiggly and spring lines correspond to the proton, \( K^- \), neutron, \( \bar{K}^0 \), Coulomb and transverse photons, respectively. The electrons run in the closed loops shown in diagrams (d) and (i). The diagrams (f) and (i) contain Coulomb ladders — the contributions with 0, 1, 2, \ldots Coulomb photons exchanged.
Modified Deser Formula:

Our modified formula, upto-and-including $O(\alpha^4, \alpha^3(m_d - m_u))$, where large nonanalytic corrections due to cusp effect are explicitly included, is best suited for the analysis of experimental data:

$$\Delta E_n^s - \frac{i}{2} \Gamma_n = -\frac{\alpha^3 \mu_c^3}{2\pi M_{K^+} + n^3} \left( T_{K^N}^{(0)} + \delta T_{K^N} \right)$$

$$\left\{ 1 - \frac{\alpha \mu_c^2 s_n(\alpha)}{4\pi M_{K^+}} T_{K^N}^{(0)} + \delta_{\text{vac}}^n \right\}$$

Coulomb Corrections

Corrections to the Deser Formula (Rough estimate):

- **Cusp Effect** $\sim 50\%$ at $O(\sqrt{\delta M})$
- **Coulomb Effects** $\sim (10$ to $15)\%$
- **Vacuum Polarization** $\sim 1\%$
- **CHPT** $\sim (-0.5 \pm 0.4)\%$ at $O(p^2)$ (or $O(\delta M)$)