Signature of the $\Lambda(1405)$ resonance in neutron spectra from the $K^- + d$ reaction

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Motivation

- the Λ(1405) plays a central role in low-energy Kaon-nuclear physics
- strong and sometimes passionate discussions about its structure
- below the $K^- p$ threshold, experimentally unreachable in two-body reactions with stable particles; only in reactions involving $n \geq 3$ particles
- the simplest case:

\[
K^- + d \rightarrow K^- + d \\
\Lambda(1405) \\
\rightarrow (\pi + \Sigma)_{I=0,1} + n \\
\rightarrow (\pi + \Sigma)_{I=1} + p
\]

- a dynamically exact calculation can be performed
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- the simplest case:

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\[ \overset{\Lambda(1405)}{\rightarrow} (\pi + \Sigma)_{I=0,1} + n \]

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- a dynamically exact calculation can be performed

What a **real** experiment can tell about the $\Lambda(1405)$?
**Method**

Coupled particle channels Faddeev-AGS treatment of the $K^- NN \leftrightarrow \pi \Sigma N$ three-body system.

The formalism has been applied to this system by several authors (Bahau & al (2003), Shevchenko & al. (2007),(2012), Ikeda & Sato (2007),(2012) and, maybe, others...) .

The main aim of these works was search for $K^- pp$ quasi-bound states or to produce reliable $K^- d$ scattering lengths.

The present work:

Exact calculation of the amplitudes of the break-up processes

$$K^- + d \rightarrow \begin{cases} 
K^- + p + n \\
\pi + \Sigma + \begin{pmatrix} p \\ n \end{pmatrix}
\end{cases} \quad A_{BU} = \langle q_{K^- p n}; \sigma_f | T_{BU} | P_{K^- \varphi d}; \sigma_i \rangle$$

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Exact -- in what sense?

In the framework of non-relativistic quantum mechanics, no approximations made in the dynamics, the output corresponds exactly to the input.

The process below \( E_{cm}^{K^-} = 50 \text{ MeV} \) is certainly basically well described within this picture. Insisting on some kind of relativization would mean to sacrifice a certain amount of dynamical accuracy...
The on-shell energy relations are:

\[
E_{cm} = \frac{E_{\pi n}^k}{E_{\pi n}^k+E_n^k} = \frac{q_{\pi n}^2}{2\mu_{\pi n}} + \frac{p_n^2}{2\mu_{n,\pi n}} + m_n + m_\pi + m_\Sigma
\]

\[
= \frac{q_{pn}^2}{2\mu_{pn}} + \frac{p_{K^-}^2}{2\mu_{K^-,pn}} + m_{K^-} + m_n + m_p
\]

\[
= E_d + \frac{P_{K^-}^2}{2\mu_{K^-,d}} + m_{K^-} + m_n + m_p
\]
The on-shell amplitude for a given neutron energy $E_n$ depends on $E_n, t$ and $\sigma_f$:

$$A(E_n, t, \sigma_f) = A_{BU}(q_{\pi\Sigma}, p_n, \sigma_f) \left| p_N \right| = \sqrt{2E_n \mu_{n,\pi\Sigma}}$$

$$\left| q_{\pi\Sigma} \right| = \sqrt{2(E_{\pi\Sigma n} - E_n)\mu_{\pi\Sigma}}$$

$$\cos(p_n, q_{\pi\Sigma}) = t$$

The neutron spectra of different possible processes are then proportional to

$$P(E_n, t, \sigma_f) \sim \frac{d\sigma}{d\Omega_{q_{\pi\Sigma}} d\Omega_{p_N} dE_n} = (2\pi)^4 \mu_{\pi\Sigma} \mu_{n,\pi\Sigma} \mu_{K^-} d \frac{q_{\pi\Sigma} p_n}{P_K} \left| A(E_n, t, \sigma_f) \right|^2$$

The inclusive neutron spectrum (when no other particles are detected) is given by

$$P(E_n) = \sum_{\sigma_f} \int_{-1}^{1} dt P(E_n, t, \sigma_f)$$
The input:

separable s-wave interactions

We have used two sets of phenomenological $\bar{K}N - \pi\Sigma$ potentials, taken from N.V. Shevchenko. One- versus two-pole $\bar{K}N - \pi\Sigma$ potential: $K^-d$ scattering length PRC 85,035203(2011),

and a more recent one

Near-threshold $K^-d$ scattering and properties of kaonic deuterium NPA 890-891,50(2012)

These potentials reproduce all known experimental data on the low-energy $\bar{K}N - \pi\Sigma$ system (the first one being fitted to the older KEK data on the kaonic hydrogen 1s level shift, while the latter one reproduces the latest SIDDHARTA values).
Here are their $\Lambda(1405)$ resonance parameters (pole positions) in MeV

<table>
<thead>
<tr>
<th></th>
<th>KEK</th>
<th>SIDDHARTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-pole</td>
<td>-23.6 – 35.7 i (1411.0 – 35.7 i)</td>
<td>-6.4 – 46.8 i (1428.1 – 46,8 i)</td>
</tr>
<tr>
<td>2-pole</td>
<td>-22.2 - 36.3 i (1412.4 – 36.3 i)</td>
<td>-14.8 – 57.2 i (1419.7 – 57.2 i)</td>
</tr>
</tbody>
</table>

Since the main issue of the work is to study the appearance of subthreshold resonances of different type in a 3-body reaction, we kept both interactions, not only the most “advanced” one.

NN potential: two-term separable potential with repulsion, reproduces the deuteron and the singlet- and triplet s-wave phase shifts
$\Sigma N$ interaction: $I_{\Sigma N} = \frac{1}{2}, \frac{3}{2}$, $S_{\Sigma N} = 1$, reproducing the scarce experimental data. In $I_{\Sigma N} = \frac{1}{2}$ state two-channel $\Sigma N - \Lambda N$.

$\pi N$ interaction neglected.

The calculation at present is restricted to $L = 0$, and to incident kaon energies up to $E_{cm}^{K^-} = 50$ MeV (kaon LAB momenta < 250 MeV/c).

For physical masses ($I = \frac{1}{2}$ and $I = \frac{3}{2}$ mixed) we have 12 unknown functions, while for averaged masses ($I = \frac{1}{2}$ only) – 8.

Numerical method: expansion of the unknown functions on cubic spline basis. Smaller matrices, no problem with logarithmic singularities – they are integrated with known functions. Also no interpolation of the solutions needed, when the break-up amplitudes are calculated.
Results

A by-product: effect of the physical masses on the $K^-d$ scattering length (fm):

<table>
<thead>
<tr>
<th></th>
<th>averaged KEK</th>
<th>averaged SIDDHARTA</th>
<th>physical KEK</th>
<th>physical SIDDHARTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-pole</td>
<td>-1.49 + 0.97 i</td>
<td>- 1.47 + 1.22 i</td>
<td>-1.52 + 0.98 i</td>
<td>- 1.50 + 1.23 i</td>
</tr>
<tr>
<td>2-pole</td>
<td>-1.57 + 1.10 i</td>
<td>- 1.50 + 1.23 i</td>
<td>-1.60 + 1.12 i</td>
<td>- 1.54 + 1.24 i</td>
</tr>
</tbody>
</table>

A few percent effect mainly in the real part. Now insignificant, may be of some use, if precise $K^-d$ atom level shifts will be available.
We have calculated the inclusive neutron spectra $P(E_n)$ for incident kaon energies both below and above the deuteron break up threshold in the range $E_{cm}^{K^{-}} = 0 - 50 \text{ MeV}$.

Overall shape – strong peak near the origin with no sign of the $\Lambda(1405)$. Above break-up: a cusp at $E_n = E_{th}$, when the $\bar{K}N$ system is at its threshold and additional neutrons from $K^- + d \rightarrow K^- + p + n$, yielding a structureless peak from $E_n = 0$ to $E_n = E_{th}$. Kinematical reason: to see the peak, $E_n$ should exceed the energy of the incident kaon by the amount of energy, which separates the pole position from the $\bar{K}N$ threshold, while in the deuteron the neutron energy distribution is dominated by the low-energy part.

Esmaili, Akaishi and Yamazaki (EAY) (PRC 83, 055207) proposed a method to – more or less – eliminate the disturbing kinematical effects in order to reveal the dynamical ones: they suggest to consider the DEViation spectrum:

$$P_{DEV} = \frac{P(E_n)}{P_{nonres}(E_n)}$$
Let’s see how this idea can be realized in our case:

Apart from the full \( K^- + d \rightarrow \pi + \Sigma + n \) break-up amplitude

\[
A_{BU} = \langle q_{\pi\Sigma} p_N; \sigma_f | T_{BU} | P_{K^-} \varphi_d ; \sigma_i \rangle
\]

we can define two approximate ones:

\[
A_{BU}^{\text{sing}} = \langle q_{\pi\Sigma} p_N; \sigma_f | t_{\pi\Sigma,KN} | P_{K^-} \varphi_d ; \sigma_i \rangle
\]

the s.c. single scattering amplitude, in which the full break-up operator \( T_{BU} \) is replaced by the two-body \( t \) -matrix \( t_{\pi\Sigma,KN} \) and the Born amplitude:

\[
A_{BU}^{\text{Born}} = \langle q_{\pi\Sigma} p_N; \sigma_f | V_{\pi\Sigma,KN} | P_{K^-} \varphi_d ; \sigma_i \rangle
\]

Kinematics in initial and final states: deuteron wave function and coordinate transformation
Thus we have three amplitudes with the properties:

\[ A_{BU} \text{ three-body dynamics + three-body kinematics} \]

\[ A_{BU}^{\text{sing}} \text{ two-body dynamics + three-body kinematics} \]

\[ A_{BU}^{\text{Born}} \text{ three-body kinematics} \]

and we expect that the DEV spectra \( P_{BU} / P_{BU}^{\text{Born}} \) and \( P_{BU}^{\text{sing}} / P_{BU}^{\text{Born}} \) will display (reveal) three- and two-body dynamics, respectively.

Our main results are displayed in the following pictures, where for a given potential and a given incident kaon energy we plotted four quantities: \( P(E_n), P_{DEV}(E_n), P_{DEV}^{\text{sing}}(E_n) \), and for comparison, the two-body \( \pi^0 \Sigma^0 \) elastic cross section corresponding to \( E_n \).
$E_{cm}^{K^-} = 1$ MeV
$E_{cm}^K = 1\text{ MeV}$
E_{cm} = 1 \text{ MeV}
$E_{K^-}^{cm} = 20 \text{ MeV}$
$E_{K^-}^{cm} = 20$ MeV
\[ E_{\text{cm}} = 20 \text{ MeV} \]
$E_{cm} = 20$ MeV

$E_{K^-} = 20$ MeV
$E_{cm} = 50$ MeV
\[ E_{cm}^\text{K^-} = 50 \text{ MeV} \]
$E_{cm}^*= 50$ MeV
$E_{cm}^*= 50$ MeV
It is expected, that the $E_n$ dependence of the born spectrum is basically determined by the initial and final states, while the details of the $V_{\bar{K}N,\pi \Sigma}$ potential influence it only weakly. This expectation is important, if the DEV spectrum method is to be applied for extracting information about $\Lambda(1405)$ from an experimentally measured neutron spectrum. Therefore we calculated $P_{\text{nonres}}(E_n)$ not with our realistic $\bar{K}N \leftrightarrow \pi \Sigma$ interactions, but with the simplest possible separable potential:

$$\left\langle q_{\pi \Sigma} \right| V^I_{\text{born}} \left| q_{\bar{K}N} \right\rangle = \frac{1}{q_{\pi \Sigma}^2 + (\beta^I_{\pi \Sigma})^2} \lambda^I_{\pi \Sigma, \bar{K}N} \frac{1}{q_{\bar{K}N}^2 + (\beta^I_{\bar{K}N})^2}$$

and took

$$\lambda^I_{\pi \Sigma, \bar{K}N} = \lambda^I_{\pi \Sigma, \bar{K}N} = 1; \beta^I_{\pi \Sigma} = \beta^I_{\pi \Sigma} = \beta^I_{\bar{K}N} = \beta^I_{\bar{K}N} = \beta_{\text{born}}$$
$E_{k^-}^{cm} = 1\ \text{MeV}$

$E_{k^-}^{cm} = 25\ \text{MeV}$
We also asked the question, under which conditions could the resonance be observed in the direct $P(E_n)$ spectra. We modified two of the $\overline{KN} \leftrightarrow \pi\Sigma$ interaction parameters for one of the potentials (KEK 1-pole) ($\lambda_{\overline{K}N,KN}^{I=0}$ and $\lambda_{\overline{K}N,\pi\Sigma}^{I=0}$) in such a way, that the position of the $\Lambda(1405)$ remained at its original place, while its width could be made smaller.

The results:
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\[ M_{\text{cc}} \]

\[ P(E_n) \]

\[ P_{\text{diff}}(E_n) \]

\[ \sigma \text{ c.m.} \]

\[ \Gamma_{\Lambda(1405)} = 50 \text{ MeV} \]

\[ \Gamma_{\Lambda(1405)} = 30 \text{ MeV} \]

\[ E_{\text{cm}}^{\text{c.m.}} = 1 \text{ MeV} \]

\[ E_{K}^{\text{c.m.}} = 25 \text{ MeV} \]
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Conclusions

• little chance to see in the inclusive spectra $P(E_n)$ the $\Lambda(1405)$ in the considered energy range; reasons: large width, kinematical effects

• DEV spectrum method eliminates the kinematical effects and reveals (in most cases) the desired maxima in the neutron spectra

• shape and position are significantly changed/shifted with respect to the original 2-body resonance; separate problem, how to deduce the parameters of the initial $\Lambda(1405)$ from a “measured” DEV spectrum; not discussed here

• 3 of the considered $KN \leftrightarrow \pi\Sigma$ potentials (KEK1, KEK2 and SIDDHARTA 2) yield neutron DEV spectra with a bump structure, which can be related to their $\Lambda(1405)$ pole positions; in the case of SIDDHARTA 1 no maxima are seen in the DEV spectra, while in single scattering approximation the resonance peak is reproduced; probable reason: extreme closeness of the resonance to the $KN$ threshold combined with its rather large width