Theoretical developments in Weak Hypernuclear Decay

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OUTLINE

✧ Weak Decay Modes of Hypernuclei
  Mesonic vs Non–Mesonic
  \( S = -1 \) and \( S = -2 \) Hypernuclei

✧ Models for Calculation
  Finite Nucleus vs Nuclear Matter in LDA

✧ Theory vs Experiment
  The Ratio \( \Gamma_n/\Gamma_p \)
  The Ratio \( \Gamma_2/\Gamma_{NM} \)
  The Decay Asymmetry for Polarized Hypernuclei

✧ Perspectives
  i) \( s \)-shell Hypernuclei and the \( \Delta I = 1/2 \) Rule
  ii) Weak Decay of \( S = -2 \) Hypernuclei

✧ Conclusions
WEAK DECAY MODES OF HYPERNUCLEI

MESONIC

\[
\begin{align*}
\Lambda & \rightarrow \pi^0 n \quad (\Gamma_{\pi^0}) \\
\Lambda & \rightarrow \pi^- p \quad (\Gamma_{\pi^-}) \\
\Gamma_M &= \Gamma_{\pi^0} + \Gamma_{\pi^-}
\end{align*}
\]

- \( Q_M = m_\Lambda - m_N - m_\pi \simeq 40 \text{ MeV} \Rightarrow p_N \simeq 100 \text{ MeV} < k_F^0 \simeq 270 \text{ MeV} \Rightarrow \) forbidden, by Pauli principle, in normal infinite nuclear matter

- Possible in finite nuclei
  
  - \( \Gamma_M \) rapidly decreases with \( A \)
  
  - \( \Gamma_M \) very sensitive to the in-medium pion behaviour \( \Rightarrow \) information on the pion–nucleus optical potential
NON–MESONIC

One–nucleon induced

\[ \Lambda n \rightarrow nn \quad \Gamma_n \]
\[ \Lambda p \rightarrow np \quad \Gamma_p \]

Two–nucleon induced

\[ \Lambda nn \rightarrow nnn \quad \Gamma_{nn} \quad \Lambda pp \rightarrow npp \quad \Gamma_{pp} \quad \Lambda np \rightarrow nnp \quad \Gamma_{np} \]

\[ \Gamma_T = \Gamma_M + \Gamma_{NM} \]
\[ \Gamma_{NM} = \Gamma_n + \Gamma_p + \Gamma_{nn} + \Gamma_{pp} + \Gamma_{np} \]
Only possible in nuclei ⇒ information on Baryon–Baryon Weak Interactions

\[ Q_{NM} = m_\Lambda - m_N \simeq 176 \text{ MeV} \implies \text{large } p_N \ (p_N \simeq 410 \text{ MeV for } 1N\text{-induced}) \]

- No Pauli blocking ⇒ \( \Gamma_{NM} > \Gamma_M \) for \( A > 5 \)

- Heavy Meson Exchange \((\pi + \rho + K + ...\) and/or Quark Exchange

Saturation of \( \Gamma_{NM} \) for increasing \( A \)

Study of \( \Gamma_n \equiv \Gamma(\Lambda n \rightarrow nn) \) and \( \Gamma_p \equiv \Gamma(\Lambda p \rightarrow np) \):
  - Spin– and Isospin–dependence (validity of the \( \Delta I = 1/2 \) isospin rule)
OTHER $S = -1$ HYPERNUCLEI ?

- $\Sigma$–Hypernuclei
  - Only $^4_2\Sigma$He exist, $V_{\Sigma}$ repulsive
  - $\Sigma N \to \Lambda N$ strong reaction prevents the observation of $\Sigma N \to NN$ weak decay

$S = -2$ HYPERNUCLEI

- $\Xi$–Hypernuclei
  - $\Xi N \to \Lambda\Lambda$ strong conversion prevents the observation of $\Xi N \to \Lambda N$ and $\Xi N \to \Sigma N$ $\Delta S = 1$ weak decays

- $\Lambda\Lambda$–Hypernuclei
  - $\Delta S = 1$ and $\Delta S = 2$ Weak decays: $\Lambda\Lambda \to \Lambda n$, $\Lambda\Lambda \to \Sigma N$, $\Lambda\Lambda \to nn$
  - Very difficult to detect: $\Gamma_{\Lambda\Lambda} \simeq \Gamma^\text{free}_{\Lambda}/(25 \div 60)$

Actual possibility of performing both Theoretical and Experimental studies on Baryon–Baryon Weak Interactions $\iff \Lambda$–Hypernuclei
Finite Nucleus: Mesonic Decay

[K. Itonaga, T. Motoba, H. Bando, NPA 489, 683 (1988)]

\[
\mathcal{H}_{\Lambda \pi N}^W = iGm_\pi^2 \bar{\psi}_N (A + B \gamma_5) \vec{r} \cdot \vec{\phi}_\pi \psi_\Lambda 
\]

\[
\Gamma^{\pi^0(\pi^-)} = c^{\pi^0(\pi^-)} (Gm_\pi^2)^2 \sum_{N > F} \int \frac{d\vec{q}}{(2\pi)^3 2\omega(\vec{q})} \frac{2\pi}{2} \delta[E_\Lambda - \omega(\vec{q}) - E_N] \times \left\{ A^2 \left| \int d\vec{r} \phi_\Lambda(\vec{r}) \phi_\pi(\vec{q}, \vec{r}) \phi_\Lambda^*(\vec{r}) \right|^2 + \frac{B^2}{4m_N^2} \left| \int d\vec{r} \phi_\Lambda(\vec{r}) \vec{\nabla} \phi_\pi(\vec{q}, \vec{r}) \phi_\Lambda^*(\vec{r}) \right|^2 \right\}
\]

✦ Shell Model \( \Lambda \) and Nucleon wave functions, \( \phi_\Lambda \) and \( \phi_N \)

✦ Pion wave function, \( \phi_\pi \), solution of the Klein–Gordon equation with proper pion–nucleus optical potential \( V_{\text{opt}} \):

\[
\left\{ \vec{\nabla}^2 - m_\pi^2 - 2\omega V_{\text{opt}}(\vec{r}) + [\omega - V_C(\vec{r})]^2 \right\} \phi_\pi(\vec{q}, \vec{r}) = 0
\]
Finite Nucleus: Non–Mesonic Decay


Shell Model Nuclear ($\Psi_R$) and Hypernuclear ($\Psi_H$) wave functions used to compute:

$$\Gamma_{n(p)} = \int \frac{dp_1}{(2\pi)^3} \int \frac{dp_2}{(2\pi)^3} 2\pi \delta (m_H - E_R - E_1 - E_2) \sum |M_{n(p)}(p_1, p_2)|^2$$

$$M_N(p_1, p_2) \equiv \langle \Psi_R; n(p_1)N(p_2) | \hat{T}_{\Lambda N \rightarrow nN} | \Psi_H \rangle$$

✧ Weak–Coupling scheme: $M_N \Rightarrow \langle nN|V_{ME}|\Lambda N \rangle$

✧ $V_{ME}$: Meson–Exchange $\Lambda N \rightarrow nN$ transition potential
  – OME: Mesons of the Pseudoscalar ($\pi$, $\eta$, $K$) and Vector ($\rho$, $\omega$, $K^*$) Octets
  – TME: uncorrelated ($2\pi$) and correlated ($2\pi/\sigma$, $2\pi/\rho$, $\pi\sigma/a_1$, $\pi\rho/a_1$)

Theoretical developments in
Weak Hypernuclear Decay (page 8)

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Finite Nucleus: Quarks

Hybrid Model: long range interactions $\implies$ hadronic degrees of freedom (OPE)
short range interactions $\implies$ 6–quark cluster model
[C.-Y. Cheung, D. P. Heddle and L. S. Kisslinger, PRC 27, 335 (1983)
[D. P. Heddle and L. S. Kisslinger, PRC 33, 608 (1986)]

Direct Quark Model combined with OME $(\pi + K + \sigma)$
[T. Inoue, M. Oka, T. Motoba and K. Itonaga, NPA 633, 312 (1998)]
[K. Sasaki, T. Inoue and M. Oka, 669, 331 (2000); NPA 678, 455(E) (2000)]

✦ Baryon–baryon short range repulsion from quark exchange between baryons
(quark antisymmetrization)

✦ Naturally includes both $\Delta I = 1/2$ and $\Delta I = 3/2$ contributions

✦ Main uncertainty from the parameterization of the effective 4–quark weak Hamiltonian
Many–Body technique for Nuclear Matter calculations, extended to finite nuclei via the LDA

Unified picture of Mesonic and Non–Mesonic decay channels, equivalent to the Finite Nucleus approach

\[ \Gamma_\Lambda = -2 \text{Im} \Sigma_\Lambda \]

\[ \Sigma_\Lambda = \Sigma_{\Lambda}^{\text{free}} + U_{L(T)} \]
\[ \Sigma_\Lambda(k) = 3i(Gm_\pi^2)^2 \int \frac{d^4q}{(2\pi)^4} \left( A^2 + \frac{B^2}{4m_N^2}q^2 \right) F_\pi^2(q)G_N(k - q)G_\pi(q) \]

\[ G_N(p) = \frac{\theta(|\vec{p}| - k_F)}{p_0 - E_N(p) - V_N + i\epsilon} + \frac{\theta(k_F - |\vec{p}|)}{p_0 - E_N(p) - V_N - i\epsilon} \]

\[ G_\pi(q) = \frac{1}{q_0^2 - \vec{q}^2 - m_\pi^2 - \Sigma^*(q)} \]

LDA

Local Fermi See of nucleons:

\[ k_F(\vec{r}) = \left\{ \frac{3}{2}\pi^2 \rho(\vec{r}) \right\}^{1/3} \]

\[ \Gamma_\Lambda(\vec{k}) = \int d\vec{r} |\psi_\Lambda(\vec{r})|^2 \Gamma_\Lambda[\vec{k}, \rho(\vec{r})] \]

\[ \Gamma_\Lambda = \int d\vec{k} |\tilde{\psi}_\Lambda(\vec{k})|^2 \Gamma_\Lambda(\vec{k}) \]
Two approaches

- Phenomenological
- Microscopic

Phenomenological Approach to the 2N

[W.M. Alberico, A. De Pace, M. Ericson and A. Molinari, PLB 256, 134 (1991)]
*A. Ramos, E. Oset and L. L. Salcedo, PRC50, 2314 (1994)]

Data on pion absorption in nuclei

* Phase space argument for the $2p2h$ configurations

Only $\Lambda np \rightarrow nnp \implies \Gamma_2 = \Gamma_{np}$

* $\Gamma_2/\Gamma_{NM} = 0.16$
Microscopic Approach to 1N and 2N

All isospin channels included in 2N: \( \Gamma_2 = \Gamma_{nn} + \Gamma_{np} + \Gamma_{pp} \)

Full quantum–mechanical approach, various many–body effects considered

[E. Bauer and F. Krmpotic, NPA 739, 109 (2004)]
[E. Bauer, NPA 818, 174 (2009)]

Fermion Antisymmetrization \( \Rightarrow \) Pauli Exchange terms

\[ \langle V_{\Lambda N \rightarrow NN} \rangle_{D+E} \quad \langle V^{NN} \rangle_{D+E} \]

[E. Bauer and G. G., NPA 828, 29 (2009)]
GSC and Fermion Antisymmetrization in 1N
[E. Bauer and G.G., PRC 81, 064315 (2010)]

Most updated results (GSC and Fermion Antisymmetrization for 1N and 2N):

\[ \frac{\Gamma_2}{\Gamma_{NM}} = 0.26 \quad \Gamma_{np} : \Gamma_{pp} : \Gamma_{nn} = 0.83 : 0.12 : 0.04 \]
Microscopic Approach to Nucleon Spectra  [Talk by Bauer]
[E. Bauer and G.G., PLB 698, 306 (2011); PLB 716, 249 (2012)]

- Quantum–mechanical approach to Nucleon FSI alternative to INC

- Key role of Interference terms

- Non–negligible role of $\Delta$–baryon resonance
  (negligible effect on decay rates)

- Discrepancies with data remain for proton emission
  (reduced with respect to INC based calculations)
Theoretical explanation of the large experimental values of \( \frac{\Gamma_n}{\Gamma_p} \equiv \frac{\Gamma(\Lambda n \to nn)}{\Gamma(\Lambda p \to np)} \) missing for decades

\[ \frac{\Gamma_n}{\Gamma_p} \]

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]

\( ^{12}_\Lambda \text{C} \) \( ^{5}_\Lambda \text{He} \)

Theory strongly underestimated Experiment!

Large uncertainties in the extraction of $\Gamma_n/\Gamma_p$ from "old" data (< year 2002)

- only Single–Proton Spectra measured $\rightarrow$ indirect determination of rates $\rightarrow$ overestimation of

$$\frac{\Gamma_n}{\Gamma_p} = \frac{\Gamma_T - \Gamma_M - \Gamma_2 - \Gamma_p}{\Gamma_p} = \Gamma_p \text{ underestimated, } \Gamma_2 \text{ neglected}$$

$$([\Gamma_2]^{\text{exp}} = 0, [\Gamma_p]^{\text{exp}} = 0.8[\Gamma_p]^{\text{th}} : [\Gamma_n/\Gamma_p]^{\text{exp}} = 1 \iff [\Gamma_n/\Gamma_p]^{\text{th}} = 0.3$$

KEK–E462/E508 ($^5\Lambda$He and $^{12}\Lambda$C):

- Nucleon–Nucleon Coincidence Spectra [2]

- more direct determination from $\frac{N_n}{N_p}$ and $\frac{N_{nn}}{N_{np}}$

$$\left[\frac{\Gamma_n}{\Gamma_p}\right]^{\text{EXP}} \sim 0.5 \quad (30\% \text{ error})$$

The One–Pion–Exchange (OPE) model predicts very small ratios:

\[
\left[ \frac{\Gamma_n}{\Gamma_p} \right]^{\text{OPE}}_{\Lambda^5\text{He}, \Lambda^{12}\text{C}} = 0.1 \div 0.2
\]

[\Delta I = 1/2 \text{ rule} + \text{strong tensor component } \Lambda N(3S_1) \to nN(3D_1) \text{ requiring } I_{nN} = 0 \iff N = p]

but reproduces the observed \( \Gamma_{\text{NM}} = \Gamma_n + \Gamma_p(\pm \Gamma_2) \)

Interaction Mechanisms beyond the OPE are responsible for the overestimation of \( \Gamma_p \) and the underestimation of \( \Gamma_n \)

Heavier Mesons (\( \rho, K, K^*, \omega, \eta, 2\pi, 2\pi/\rho, 2\pi/\sigma \)) [Parreño et al., Itonaga et al., Jido et al., Krmpotic et al.]

Direct Quark Mechanism [Oka et al.]

Two–Nucleon Induced Mechanism [Alberico et al., Ramos et al., Bauer et al.]

Nucleon Final State Interactions [Ramos et al., Garbarino et al., Bauer et al.]
Including **Heavy Meson Exchange** [1] and **Direct Quark** [2] contributions

\[
\begin{bmatrix} \Gamma_n \\ \Gamma_p \end{bmatrix}^{\text{TH}} = 0.3 - 0.7
\]


The determination of \(\Gamma_n/\Gamma_p\) from \(N_{nn}/N_{np}\) Data requires Theoretical Analyses [3]:

✦ **Two–Nucleon Induced Decays**

✦ **Nucleon FSI (INC)**

\[
\begin{bmatrix} N_{nn} \\ N_{np} \end{bmatrix}^{\text{KEK}} \sim 0.5 \quad \Rightarrow \quad \begin{bmatrix} \Gamma_n \\ \Gamma_p \end{bmatrix}^{\text{“EXP”}} \sim 0.4 \quad \text{(30\% error)}
\]

convincing evidence for a **SOLUTION OF THE PUZZLE**

Table 1: Theory vs Experiment for the 2N in $^{12}_\Lambda$C

<table>
<thead>
<tr>
<th>Authors</th>
<th>Model/Experiment</th>
<th>$\Gamma_2/\Gamma_{NM}$</th>
<th>$\Gamma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramos, Oset, Salcedo (1994)</td>
<td>Phen. $(\Gamma_2 = \Gamma_{np})$</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td>Alberico, De Pace, G., Ramos (2000)</td>
<td>Phen. $(\Gamma_2 = \Gamma_{np})$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>G., Parreño, Ramos (2003)</td>
<td>Phen. $(\Gamma_2 = \Gamma_{np})$</td>
<td>0.20</td>
<td>0.19</td>
</tr>
<tr>
<td>Bauer, Krmpotic (2004)</td>
<td>Micr.</td>
<td>0.24</td>
<td>0.51</td>
</tr>
<tr>
<td>Bauer, G. (2009)</td>
<td>Micr. + GSN</td>
<td>0.37</td>
<td>0.31</td>
</tr>
<tr>
<td>Bauer, G. (2010)</td>
<td>Micr. + GSC$(1N,2N)$</td>
<td>0.34</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>Micr. + GSC$(1N,2N)$ + Ant.</td>
<td>0.26</td>
<td>0.25</td>
</tr>
<tr>
<td>Kim et al. (2009)</td>
<td>KEK</td>
<td>0.29 ± 0.13</td>
<td>0.27 ± 0.13</td>
</tr>
<tr>
<td>Agnello et al. (2010)</td>
<td>FINUDA $(A = 5-16)$</td>
<td>0.24 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>Agnello et al. (2011)</td>
<td>FINUDA $(A = 5-16)$</td>
<td>0.21 ± 0.07</td>
<td></td>
</tr>
</tbody>
</table>

♦ Forthcoming Measurement by E18@J–PARC [Talk by Outa]

GSN = Ground State Normalization
GSC = Ground State Correlations
Ant. = Fermion Antisymmetrization ↔ Pauli Exchange terms
The non–mesonic rates of $^{12}_Λ C$ in the most updated Microscopic approach
[E. Bauer and G.G., PRC 81, 064315 (2010)]

- GSC: opening of 2N $\Rightarrow$ sizable increase of $\Gamma_{NM}$
- Ant.: increase of $\Gamma_1$, decrease of $\Gamma_2$, important for reproducing $\Gamma_2/\Gamma_{NM}$

<table>
<thead>
<tr>
<th>Ant./GSC</th>
<th>$\Gamma_n$</th>
<th>$\Gamma_p$</th>
<th>$\Gamma_1$</th>
<th>$\Gamma_2$</th>
<th>$\Gamma_{NM}$</th>
<th>$\Gamma_n/\Gamma_p$</th>
<th>$\Gamma_2/\Gamma_{NM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No/No</td>
<td>0.15</td>
<td>0.47</td>
<td>0.62</td>
<td>0</td>
<td>0.62</td>
<td>0.31</td>
<td>0</td>
</tr>
<tr>
<td>Yes/No</td>
<td>0.18</td>
<td>0.56</td>
<td>0.74</td>
<td>0</td>
<td>0.74</td>
<td>0.33</td>
<td>0</td>
</tr>
<tr>
<td>No/Yes</td>
<td>0.15</td>
<td>0.47</td>
<td>0.61</td>
<td>0.31</td>
<td>0.91</td>
<td>0.31</td>
<td>0.34</td>
</tr>
<tr>
<td>Yes/Yes</td>
<td>0.19</td>
<td>0.55</td>
<td>0.73</td>
<td>0.25</td>
<td>0.98</td>
<td>0.34</td>
<td>0.26</td>
</tr>
<tr>
<td>KEK</td>
<td>0.23 ± 0.08</td>
<td>0.45 ± 0.10</td>
<td>0.68 ± 0.13</td>
<td>0.27 ± 0.13</td>
<td>0.95 ± 0.04</td>
<td>0.51 ± 0.13 ± 0.05</td>
<td>0.29 ± 0.13</td>
</tr>
<tr>
<td>FINUDA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>FINUDA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.21 ± 0.07</td>
</tr>
</tbody>
</table>
Figure 1: Data for $^{11}\Lambda\text{B}$, $^{12}\Lambda\text{C}$, $^{27}\Lambda\text{Al}$, $^{28}\Lambda\text{Si}$, $^{56}\Lambda\text{Fe}$ from KEK and for $A = 180-220$ from COSY. Datum for $\Gamma_2$ from KEK. From [E. Bauer and G.G., PRC 81, 064315 (2010)]
\[ \pi^+ n \rightarrow \Lambda K^+ \]

\[ p_\Lambda = \Lambda \text{ Polarization} \]

\[ a_\Lambda = \text{Intrinsic } \Lambda \text{ Asymmetry Parameter} \]

\[ a_\Lambda \leftrightarrow \text{Interference among PC and PV } \bar{\Lambda}p \rightarrow np \text{ channels} \]

\[ \Rightarrow \text{information on strengths and relative phases of the decay amplitudes} \]
<table>
<thead>
<tr>
<th>Model/Experiment</th>
<th>$^5\Lambda$He</th>
<th>$^{12}\Lambda$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sasaki et al. (2002)</td>
<td></td>
<td>−0.68</td>
</tr>
<tr>
<td>$\pi + K + DQ$</td>
<td></td>
<td>−0.68</td>
</tr>
<tr>
<td>Parreño et al. (2002)</td>
<td></td>
<td>−0.73</td>
</tr>
<tr>
<td>OME = $\pi + \rho + K + K^* + \omega + \eta$</td>
<td></td>
<td>−0.54</td>
</tr>
<tr>
<td>Barbero et al. (2005)</td>
<td></td>
<td>−0.53</td>
</tr>
<tr>
<td>OME = $\pi + \rho + K + K^* + \omega + \eta$</td>
<td></td>
<td>−0.54</td>
</tr>
<tr>
<td>Alberico et al. (2005)</td>
<td></td>
<td>−0.46</td>
</tr>
<tr>
<td>OME + FSI (INC)</td>
<td></td>
<td>−0.37</td>
</tr>
<tr>
<td>Chumillas et al. (2007)</td>
<td></td>
<td>+0.028</td>
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<tr>
<td>OME + 2$\pi$ + 2$\pi/$σ + FSI (INC)</td>
<td></td>
<td>−0.126</td>
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<tr>
<td>Itonaga et al. (2007, 2010)</td>
<td></td>
<td>+0.083</td>
</tr>
<tr>
<td>$\pi + K + \omega + 2\pi/$ρ + 2$\pi/$σ + $\rho\pi/a_1$ + $\sigma\pi/a_1$</td>
<td></td>
<td>+0.045</td>
</tr>
<tr>
<td>Bauer et al. (2012)</td>
<td></td>
<td>+0.037</td>
</tr>
<tr>
<td>OME + 2$\pi$ + 2$\pi/$σ + 2N + FSI (Micr.)</td>
<td></td>
<td>−0.16 ± 0.28^{+0.18}_{−0.00}</td>
</tr>
<tr>
<td>KEK–E508</td>
<td>+0.07 ± 0.08^{+0.08}_{−0.00}</td>
<td>−0.16 ± 0.28^{+0.18}_{−0.00}</td>
</tr>
<tr>
<td>KEK–E462</td>
<td></td>
<td>−0.16 ± 0.28^{+0.18}_{−0.00}</td>
</tr>
</tbody>
</table>

OME + TPE: Agreement with rate and asymmetry data in $^5\Lambda$He and $^{12}\Lambda$C!
Effective Field Theory with $\pi + K^+$ Leading–Order Contact Interactions
[A. Parreño, C. Bennhold and B.R. Holstein, PRC70, 051601 (2004)]

Fit of experimental rates and asymmetries for $^5_\Lambda$He and $^{12}_\Lambda$C

$\implies$ Dominance of Spin– and Isospin–Independent contact terms

$\implies$ Importance of the Scalar–Isoscalar channel ($\sigma, 2\pi, 2\pi/\sigma, ...$)

Interpretation of OME models in terms of the EFT LECs
[Talk by Pérez–Obiol]
Experiment and Theory now agree on decay rates and asymmetries.

Diagram showing the ratio of decay rates and angmoments for different nuclei.

OME + TPE reproduces all data (no exotic mechanisms and $\Delta I = 3/2$ contributions).

However, improvements are necessary to achieve a detailed understanding of the Non–Mesonic Weak Decay reaction mechanisms.

- Still model–dependent results in OME and DQ calculations (unknown weak meson–baryon–baryon couplings, validity of $\Delta I = 1/2$ rule)
- Extend the study to $S = -2$ hypernuclei
i) s–shell Hypernuclei and the $\Delta I = 1/2$ Rule

- Block–Dalitz Phenomenological Model $\implies$ Spin–Isospin structure of $\Lambda N \to nN$
- Relations which test the $\Delta I = 1/2$ Rule

\[
\frac{\Gamma_n(4\Lambda \text{He})}{\Gamma_p(4\Lambda \text{H})} = \frac{\Gamma_n(4\Lambda \text{H})\Gamma_n(4\Lambda \text{He})}{\Gamma_p(5\Lambda \text{He})} = \frac{R_{n0}}{R_{p0}} \iff \Delta I = 1/2 \text{ Rule: } \frac{R_{n0}}{R_{p0}} = 2
\]

- By using the most recent KEK data for $\Gamma_{NM}(5\Lambda \text{He})$ and $\frac{\Gamma_n(5\Lambda \text{He})}{\Gamma_p(5\Lambda \text{He})}$:

\[
\Delta I = 1/2 \text{ rule} \quad \text{Experiment}
\]

\[
\Gamma_{NM}(4\Lambda \text{He}) = 0.25^{+0.04}_{-0.01} \iff 0.177 \pm 0.028 \quad \text{(BNL–E788)}
\]

\[
\Gamma_{NM}(4\Lambda \text{H}) = 0.08^{+0.03}_{-0.02} \iff 0.17 \pm 0.11 \quad \text{(KEK)}
\]

$\implies$ violation of the $\Delta I = 1/2$ rule? Too early to conclude!

- E22@J–PARC: precise measurement of $\Gamma_n$ and $\Gamma_p$ for $4\Lambda \text{H}$ and $4\Lambda \text{He}$
ii) Weak Decay of $S = -2$ Hypernuclei

✦ ΛΛ Hypernuclei allow Hyperon–Induced Non–Mesonic Weak Decay

- $\Lambda\Lambda \rightarrow \Lambda n \quad \Lambda\Lambda \rightarrow \Sigma^0 n \quad \Lambda\Lambda \rightarrow \Sigma^- p \quad (\Delta S = 1)$
- $\Lambda\Lambda \rightarrow nn \quad (\Delta S = 2)$
- $\Gamma_{\Delta S=1}(^6\Lambda\Lambda\text{He})/\Gamma_\Lambda = 0.017 \ [1], \ 0.026 \ [2], \ 0.040 \ [3]$


✦ KEK–E373: NAGARA event \[H. Takahashi et al., PRL 87, 212502 (2001)\]
- Production of $^6\Lambda\Lambda\text{He}$ (ΛΛ interaction is weakly attractive)
- Observation of a Weak Decay to $\Sigma^- p$ ($BR^{\text{exp}} \simeq 0.01$)
  \[T. Watanabe et al., EPJA 33, 265 (2007)\]

$$\Lambda\Lambda \rightarrow \Sigma^- p \quad \text{(but } BR^{\text{th}} \simeq 0.001)$$

$$H(uuddss) \rightarrow \Sigma^- p \quad (BR^{\text{th}} \simeq 0.01)$$

[J.F. Donoghue, E. Golowich and B.R. Holstein, PRD 34, 3434 (1986)]

$$\Lambda\Lambda \rightarrow H \rightarrow \Sigma^- p$$

✦ New experiments and systematic theoretical studies needed
**CONCLUSIONS**

- Agreement between Experiment and Theory on Decay Rates ($\Gamma_{NM}$, $\Gamma_2/\Gamma_{NM}$ and $\Gamma_n/\Gamma_p$) and Asymmetries is reasonable

\[
\Gamma_{NM}^{(5\Lambda\text{He})} \sim 0.4 \quad \Gamma_{NM}^{(12\Lambda\text{C})} \sim 0.9 \quad (\text{EXP: few \% error})
\]

\[
\frac{\Gamma_n}{\Gamma_p}^{(5\Lambda\text{He})} \sim \frac{\Gamma_n}{\Gamma_p}^{(12\Lambda\text{C})} \sim 0.5 \quad (\text{EXP: 30\% error}) \quad (\text{TH: Heavy Meson Exchange})
\]

\[
\frac{\Gamma_2}{\Gamma_{NM}}^{(12\Lambda\text{C})} \sim 0.25 \quad (\text{EXP: 40\% error}) \quad (\text{TH: GSC and Ant.})
\]

\[
a_n^{(5\Lambda\text{He})} \sim 0 \div 0.2 \quad a_n^{(12\Lambda\text{C})} \sim -0.1 \div +0.3 \quad (\text{EXP: > 100\% error}) \quad (\text{TH: Scalar Isoscalar Exchange})
\]

- Improved/New models/measurements for a detailed understanding of the Non–Mesonic Weak Decay reaction mechanisms
  - $s$–shell Hypernuclei and the $\Delta I = 1/2$ rule
  - Non–Mesonic Weak Decay of $S = -2$ Hypernuclei
  - Future Experiments: J–PARC, PANDA@FAIR
ADDITIONAL SLIDES
RESULTS


Theoretical developments in Weak Hypernuclear Decay (page 31)
Theoretical developments in Weak Hypernuclear Decay

Theoretical developments in Weak Hypernuclear Decay

Figure 2: Single–proton kinetic energy spectra per NMWD of $^{12}_{\Lambda}$C.
Theoretical developments in Weak Hypernuclear Decay

Figure 3: Kinetic energy correlations of np pairs emitted per NMWD of $^{12}_ΛC$
Theoretical developments in Weak Hypernuclear Decay

G. Garbarino
Torino

Figure 4: Angular distribution of $nn$, $np$ and $pp$ pairs emitted per NMWD of $^{12}_ΛC$.
Number of primary $nn$ and $np$ pairs:

$$N_{nn}^{wd} \propto \Gamma_n \quad N_{np}^{wd} \propto \Gamma_p$$

Denoting with $N_{nn}$ and $N_{np}$ the number of nucleons emitted by the nucleus:

$$\frac{\Gamma_n}{\Gamma_p} \equiv \frac{\Gamma(\Lambda n \to nn)}{\Gamma(\Lambda p \to np)} \equiv \frac{N_{nn}^{wd}}{N_{np}^{wd}} \neq \frac{N_{nn}}{N_{np}} = R_2 (\Gamma_2, \text{FSI})$$

<table>
<thead>
<tr>
<th></th>
<th>$^5\Lambda$He</th>
<th>$^5\Lambda$He</th>
<th>$^{12}\Lambda$C</th>
<th>$^{12}\Lambda$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPE</td>
<td></td>
<td>0.25</td>
<td>0.09</td>
<td>0.24</td>
</tr>
<tr>
<td>OME</td>
<td></td>
<td>0.51</td>
<td>0.34</td>
<td>0.39</td>
</tr>
<tr>
<td>KEK–E462</td>
<td>0.45 ± 0.11 ± 0.03</td>
<td></td>
<td></td>
<td>0.40 ± 0.10</td>
</tr>
<tr>
<td>KEK–E508</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data from B. H. Kang et al., PRL 96, 062301 (2006); M. J. Kim et al., PLB 641, 28 (2006); H. Outa, NPA 754, 157c (2005)
A weak–decay–model independent analysis of $\Gamma_n/\Gamma_p$

- Total number of $NN$ pairs emitted per NMWD:

  \[
  N_{nn} = \frac{N_{nn}^{1Bn} \Gamma_n + N_{nn}^{1Bp} \Gamma_p + N_{nn}^{2B} \Gamma_2}{\Gamma_n + \Gamma_p + \Gamma_2} \\
  N_{np} = \frac{N_{np}^{1Bn} \Gamma_n + N_{np}^{1Bp} \Gamma_p + N_{np}^{2B} \Gamma_2}{\Gamma_n + \Gamma_p + \Gamma_2}
  \]

  which define the six weak–decay–model independent quantities: $N_{nn}^{1Bn}$ (the number of $nn$ pairs emitted per neutron–induced NMWD), etc.

- From a measurement of $N_{nn}/N_{np}$ and appropriate values for $\Gamma_2/\Gamma_1$:

  \[
  \frac{\Gamma_n}{\Gamma_p} = \frac{N_{nn}^{1Bp} + N_{nn}^{2B} \Gamma_2 \Gamma_1}{\left( N_{np}^{1Bp} + N_{np}^{2B} \Gamma_2 \Gamma_1 \right) \frac{N_{nn}}{N_{np}} - N_{nn}^{1Bn} - N_{nn}^{2B} \Gamma_2 \Gamma_1} - \left( N_{np}^{1Bp} + N_{np}^{2B} \Gamma_2 \Gamma_1 \right) \frac{N_{nn}}{N_{np}}
  \]

- From KEK data we obtained:

  \[
  ^5\Lambda\text{He} \hspace{1cm} \frac{\Gamma_n}{\Gamma_p} = 0.26 \pm 0.11 \hspace{1cm} \Gamma_2 = 0.20 \Gamma_1 \hspace{1cm} (\frac{\Gamma_n}{\Gamma_p} = 0.39 \pm 0.11 \hspace{1cm} \Gamma_2 = 0) \\
  ^{12}\Lambda\text{C} \hspace{1cm} \frac{\Gamma_n}{\Gamma_p} = 0.29 \pm 0.14 \hspace{1cm} \Gamma_2 = 0.25 \Gamma_1 \hspace{1cm} (\frac{\Gamma_n}{\Gamma_p} = 0.38 \pm 0.14 \hspace{1cm} \Gamma_2 = 0)
  \]
Exp–Th disagreement on Proton Spectra

Agreement for Neutrons

$^{12}$C

Disagreement for Protons

$^{\Lambda}$ He

BNL–E788: Neutron and Proton Spectra for $^{4}_{\Lambda}$He

[J. D. Parker et al., PRC 76, 035501 (2007)]

FINUDA: Proton Spectra for $^{5}_{\Lambda}$He to $^{16}_{\Lambda}$O:
peaking structure at $\simeq 80$ MeV

[M. Agnello et al., NPA 804, 151 (2008)]
Comparison with theoretical calc. for angular correlation

**Experimental Data**

- $^5\Lambda$He (E462)
- $^12\Lambda$C (E508)

**Theoretical Calc.**

- Garbarino's calc.

Assuming $G_n/G_p = 0.46$ (for $^5\Lambda$He), 0.34 (for $^12\Lambda$C)

Considered 2N-induced (≤ 20%), FSI

Asymmetry: OME + Nucleon FSI


OME = $\pi + \rho + K + K^* + \eta + \omega$

$I(\theta) = I_0 \left[1 + p_\Lambda a_\Lambda \cos \theta\right] \quad I^M(\theta) = I^M_0 \left[1 + p_\Lambda a^M_\Lambda \cos \theta\right]$

<table>
<thead>
<tr>
<th></th>
<th>$^5_\Lambda$He</th>
<th>$^{11}_\Lambda$B</th>
<th>$^{12}_\Lambda$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\Lambda$</td>
<td>$-0.68$</td>
<td>$-0.81$</td>
<td>$-0.73$</td>
</tr>
<tr>
<td>$a^M_\Lambda (T_p \geq 30 \text{ MeV})$</td>
<td>$-0.46$</td>
<td>$-0.39$</td>
<td>$-0.37$</td>
</tr>
<tr>
<td>$a^M_\Lambda (T_p \geq 50 \text{ MeV})$</td>
<td>$-0.52$</td>
<td>$-0.55$</td>
<td>$-0.51$</td>
</tr>
<tr>
<td>$a^M_\Lambda (T_p \geq 70 \text{ MeV})$</td>
<td>$-0.55$</td>
<td>$-0.70$</td>
<td>$-0.65$</td>
</tr>
</tbody>
</table>

KEK–E462 \hspace{1cm} $0.07 \pm 0.08^{+0.08}_{-0.00}$

KEK–E508 \hspace{1cm} $-0.16 \pm 0.28^{+0.18}_{-0.00}$

Data from [T. Maruta et al., EPJA 33, 255 (2007)]
Effective Field Theory: $\pi + K+$ Leading–Order Contact Interactions

[A. Parreño, C. Bennhold and B. R. Holstein, PRC 70, 051601 (2004)]

- LOCI coefficients fixed to reproduce experimental $\Gamma_{NM}$ and $\Gamma_n/\Gamma_p$ for $^5\Lambda$He, $^{11}\Lambda$B and $^{12}\Lambda$C and $a_\Lambda(^5\Lambda$He)

- Predicted a dominating Central, Spin– and Isospin–Independent contact term

$\pi + K + \sigma +$ Direct Quark

[K. Sasaki, M. Izaki, M. Oka, PRC 71, 035502 (2005)]

- Decay data for $s$-shell hypernuclei fitted to obtain the weak couplings of the Scalar–Isoscalar $\sigma$–meson, $H_{W\Lambda\sigma N} = g_W \bar{\psi}_N (A_\sigma + B_\sigma \gamma_5) \phi_\sigma \psi_\Lambda$

- All $^5\Lambda$He decay observables reasonably reproduced. No calculation for $^{12}\Lambda$C

OME $+$ $\sigma$, OME $= \pi + \rho + K + K^* + \eta + \omega$

[C. Barbero and A. Mariano, PRC 73, 024309 (2006)]

- Unknown $\sigma$ couplings fixed to reproduce measured $\Gamma_{NM}(^5\Lambda$He) and $\Gamma_n/\Gamma_p(^5\Lambda$He)

- Improved overall agreement with experiment for $^{12}\Lambda$C and $^5\Lambda$He but data for $a_\Lambda(^5\Lambda$He) could not be reproduced

Importance of the Scalar–Isoscalar channel in Asymmetry calculations
One–Meson–Exchange + Two–Pion–Exchange


✦ Uncorrelated ($2\pi$) and Correlated ($2\pi/\sigma$) Two–Pion–Exchange (TPE)

[D. Jido, E. Oset and J.E. Palomar, NPA 694, 525 (2001)]

✦ $2\pi/\sigma$ motivated by Chiral Unitary Theory

$\Lambda\Lambda$ $N\ NN$ $\pi\ \pi$

$\Lambda\NN$ $\pi$ $\pi$

$W\ S$ $S$ $\pi$

$\Delta, N$ $\Delta, N$ $\Delta, N$

$\Delta, N + \Delta, N$

$W\ S$

(a) (b) (c)

✦ $2\pi$: dominated by the isoscalar channel

✦ $2\pi/\sigma$ reproduces $\pi\pi$ scattering data in the scalar sector

✦ No Free Parameter: couplings determined from chiral meson–meson and meson–baryon Lagrangians
<table>
<thead>
<tr>
<th>Model</th>
<th>$\Gamma_{NM} = \Gamma_n + \Gamma_p$</th>
<th>$\frac{5\Lambda}{\Lambda}$ He $\Gamma_n/\Gamma_p$</th>
<th>$a_\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OME</td>
<td>0.379</td>
<td>0.474</td>
<td>$-0.590$</td>
</tr>
<tr>
<td>OME+TPE</td>
<td>0.388</td>
<td>0.415</td>
<td>$+0.041$</td>
</tr>
<tr>
<td>OME+TPE+FSI</td>
<td></td>
<td></td>
<td>$+0.028$</td>
</tr>
<tr>
<td>KEK–E462</td>
<td>$0.424 \pm 0.024$</td>
<td>$0.40 \pm 0.11$ ($1N$)</td>
<td>$+0.07 \pm 0.08^{+0.08}_{-0.00}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.27 \pm 0.11$ ($1N + 2N$)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Gamma_{NM} = \Gamma_n + \Gamma_p$</th>
<th>$\frac{12\Lambda}{\Lambda}$ C $\Gamma_n/\Gamma_p$</th>
<th>$a_\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OME</td>
<td>0.667</td>
<td>0.357</td>
<td>$-0.698$</td>
</tr>
<tr>
<td>OME+TPE</td>
<td>0.722</td>
<td>0.366</td>
<td>$-0.207$</td>
</tr>
<tr>
<td>OME+TPE+FSI</td>
<td></td>
<td></td>
<td>$-0.126$</td>
</tr>
<tr>
<td>KEK–E508</td>
<td>$0.940 \pm 0.035$</td>
<td>$0.38 \pm 0.14$ ($1N$)</td>
<td>$-0.16 \pm 0.28^{+0.18}_{-0.00}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0.29 \pm 0.14$ ($1N + 2N$)</td>
<td></td>
</tr>
<tr>
<td>KEK–E307</td>
<td>0.828 $\pm$ 0.087</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

✧ Moderate change of the Decay Rates, huge influence on the Asymmetries!
✧ Agreement with both Asymmetry and Decay Rate data for both $\frac{5\Lambda}{\Lambda}$ He and $\frac{12\Lambda}{\Lambda}$ C!
<table>
<thead>
<tr>
<th></th>
<th>(^{5}_{\Lambda}\text{He} \rightarrow \text{OME} \rightarrow \text{OME} + \text{TPE}</th>
<th>\text{OME}</th>
<th>\text{OME} + \text{TPE}</th>
<th>\text{OME}</th>
<th>\text{OME} + \text{TPE}</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(^{1}\text{S}_0 \rightarrow ^{1}\text{S}_0)</td>
<td>−0.1044</td>
<td>+0.0835</td>
<td>(\text{AE})</td>
<td>−0.2854</td>
</tr>
<tr>
<td>B</td>
<td>(^{1}\text{S}_0 \rightarrow ^{3}\text{P}_0)</td>
<td>+0.0057</td>
<td>+0.0057</td>
<td>(\text{BC})</td>
<td>+0.0027</td>
</tr>
<tr>
<td>C</td>
<td>(^{3}\text{S}_1 \rightarrow ^{3}\text{S}_1)</td>
<td>−0.1399</td>
<td>+0.1480</td>
<td>(\text{BD})</td>
<td>−0.0029</td>
</tr>
<tr>
<td>D</td>
<td>(^{3}\text{S}_1 \rightarrow ^{3}\text{D}_1)</td>
<td>−0.1814</td>
<td>−0.1814</td>
<td>(\text{CF})</td>
<td>−0.0856</td>
</tr>
<tr>
<td>E</td>
<td>(^{3}\text{S}_1 \rightarrow ^{1}\text{P}_1)</td>
<td>+0.3833</td>
<td>+0.3833</td>
<td>(\text{DF})</td>
<td>−0.2186</td>
</tr>
<tr>
<td>F</td>
<td>(^{3}\text{S}_1 \rightarrow ^{3}\text{P}_1)</td>
<td>+0.2234</td>
<td>+0.2234</td>
<td>(\alpha_{\Lambda})</td>
<td>−0.590</td>
</tr>
</tbody>
</table>

\[\Gamma_p = \sum_{\alpha=A,\ldots,F} |\alpha|^2 \]

0.257

0.275

\(a_{\Lambda}\)

0.590

0.041

- Spectroscopic notation: \(\Lambda p (^{2S+1}L_J) \rightarrow np (^{2S'+1}L'_J)\)

- OME \rightarrow OME + TPE:
  - Drastic change of the Scalar–Isoscalar amplitudes \(A\) and \(C\)
  - \(AE\) interference changes sign and cancels the \(DF\) contribution
Perspectives: “Exotic” Hypernuclei

✦ Neutron– and Proton–Rich ($^6_\Lambda$H, $^9_\Lambda$He; $^7_\Lambda$Be, $^8_\Lambda$C)
  – $\Gamma_n/\Gamma_p$ for extreme $N/Z$
  – Effects of (low–density) Neutron and Proton Halos on NMWD
  – Present and Future searches:
    KEK and FINUDA: formation probability studies (upper limits)
    HypHI@GSI: in–flight decays, no surrounding target ($T_N^{th} \to 0$)
    J–PARC: E10
    Nuclotron@JINR (Dubna): relativistic hypernuclei

✦ Medium and Heavy: $A > 11$ (saturation property of $\Gamma_{NM}$)
  – KEK: saturation at $\Gamma_{NM}(^{28}_\Lambda$Si $- ^{56}_\Lambda$Fe) $\simeq 1.2$, in agreement with Theory
  – COSY–13@Juelich: $p + A$, $A = $ Au, Bi and U targets, measurement of fragments from fission induced by NMWD, no direct identification of hypernuclear formation $\Gamma_{NM}(A \simeq 180 - 225) = 1.81 \pm 0.14$
  – CEBAF@JLAB: proposal for high–precision measurement of lifetime of heavy hypernuclei?