Introduction

Multi-hadrons on the lattice -- the reason why it is so hard --

BB Forces from LQCD

Summary

Strangeness Nuclear Physics from Lattice QCD

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T. Hatsuda (Nishina Center, RIKEN)
[1] Introduction

   -- the reason why it is so hard --

[3] BB Forces from LQCD

[4] Summary
Introduction

hadron

1 fm

nuclei

10 fm

neutron star

10 km
HAL QCD Strategy: From QCD to Compact stars

BG/L -> PACS-CS -> T2K -> BG/Q -> KEI
(10TF -> 100TF -> 1PF -> 10PF)

Neutron star:
- max mass, cooling etc

Supernova explosion

Hypernuclei
- Neutron star merger
- max mass, cooling etc

J-PARC (KEK/JAEA)

KEI Computer @ AICS (RIKEN)
(10PFlops)
Advanced Institute for Computational Science (AICS), RIKEN
10 PFlops supercomputer KEI “京” (full operation started on Sep.28, 2012)


Five “strategic” programs (FY 2010-2015)

1. Life and Medicine  
2. New Materials  
3. Environment  
4. Engineering  
5. Particle, Nuclear and Astrophysics

Project 1:  Baryon-Baryon interaction from lattice QCD simulations at physical point
Project 2:  Large scale quantum many-body calculation of nuclei and its applications
Project 3:  Realistic simulation of supernova explosion and black-hole formation
Project 4:  Large scale simulation of first generation of stars and galaxies

Physical point simulation started : 96^4 lattice, a=0.1fm, L=9.6fm, m_\pi=135MeV
Quantum Chromo Dynamics

\[
\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q} \gamma^\mu (i\partial_\mu - gt^a A^a_\mu)q - m\bar{q}q
\]

\[G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu\]

Running masses: \( m_q(Q) \)

<table>
<thead>
<tr>
<th>quark masses (from lattice QCD)</th>
<th>[MeV] (MS-bar @ 2GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_u )</td>
<td>2.19(15)</td>
</tr>
<tr>
<td>( m_d )</td>
<td>4.67(20)</td>
</tr>
<tr>
<td>( m_s )</td>
<td>94(3)</td>
</tr>
</tbody>
</table>

FLAG working group,
arXiv:1011.4408 [hep-lat]

Running coupling: \( \alpha_s(Q) = \frac{g^2}{4\pi} \)

- \( N_f=2+1 \) on the lattice

World average (w/o lattice): 0.1186(11)

\( \alpha_s(M_Z) = 0.1189(4^+6) \)
Physical point simulation in (2+1)-flavor QCD (by PACS-CS Coll.)

Improved Wilson + Iwasaki gauge action; \( a = 0.09 \) fm, \( L=2.9 \) fm, \( m_\pi =135 \) MeV

Baryon force: From phenomenology to 1st principle

- **NN int.:** about 4500 np and pp scatt. data

<table>
<thead>
<tr>
<th>“high precision” NN interactions</th>
<th># of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD Bonn (p space)</td>
<td>38</td>
</tr>
<tr>
<td>AV18 (r space)</td>
<td>40</td>
</tr>
<tr>
<td>EFT in N³LO (nπ+contact)</td>
<td>24</td>
</tr>
</tbody>
</table>


- NNN, YN, YY: data very limited
- YNN, YYN, YYY: none

QCD has only four parameters: $m_u, m_d, m_s, \Lambda_{QCD}$
Multi-hadrons on the Lattice
Multi-hadron Dilemma

\[ E \]

- Resonance region
- Scattering region

\( NN\pi \)

\( NN \)
$E_{th} \sim 135$ MeV

$\Delta E \sim 16 \left(\frac{3.2}{L}\right)^2$ MeV
Multi-hadron Dilemma

\[ E_{th} \sim 135 \text{ MeV} \]

\[ \Delta E \sim 16 \left(\frac{3.2}{L}\right)^2 \text{ MeV} \]

\[ e^{-Ht} = \sum_{n} e^{-E_n t} |n\rangle \langle n| \]

Resonance region

Scattering region

\[ 0 \]

\[ \frac{1}{E_{th}} \sim 1.5 \text{ fm} \]

\[ \frac{1}{\Delta E} \sim 12 \left(\frac{L}{3.2}\right)^2 \text{ fm} \]

\[ \frac{S}{N} \sim \sqrt{N} \exp\left[-2(m_N - 1.5m_\pi)t\right] \]
Solution of the Dilemma: Interaction kernel (=non-local potential)

Resonance region

$1/E_{th}$

$\sim 1.5$ fm

Scattering region

$1/(\Delta E)$

$\sim 12 (L/3.2)^2$ fm
Solution of the Dilemma: Interaction kernel (=non-local potential)

\[ \phi_n(\vec{r}) = \langle 0| N(\vec{x} + \vec{r}) N(\vec{x}) | n \rangle \]

\[ (k_n^2 + \nabla^2) \phi_n(\vec{r}) = \int U(\vec{r}, \vec{r}') \phi_n(\vec{r}') d^3r' \]

\[ \phi(\vec{r}, t) = \sum_{n \leq n_{th}} \phi_n(\vec{r}) e^{-E_n t} \]

\[ \left( -\left( \frac{1}{2} \partial_t \right)^2 - m_N^2 + \nabla^2 \right) \phi(\vec{r}, t) = \int U(\vec{r}, \vec{r}') \phi(\vec{r}', t) d^3r' \]

Ishii, Aoki & Hatsuda,
PRL 99 (2007) 022001
PTP 123 (2010) 89

Ishii et al. [HAL QCD Coll.],

Resonance region

Scattering region

\[ 0 \]

\[ 1/E_{th} \sim 1.5 \text{ fm} \]

\[ 1/(\Delta E) \sim 12 \left( L/3.2 \right)^2 \text{ fm} \]
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\[ U(\vec{r}, \vec{r'}) = m_N V(\vec{r}, \nabla)\delta(\vec{r} - \vec{r'}) \]

\[ V(\vec{r}, \nabla) = V_C(r) + S_{12} V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{ V_D(r), \nabla^2 \} + \ldots \]

- "Potential" is not an observable but is a nice tool to calculate observables
- "Potential" is volume insensitive (i.e. Lattice Friendly)
Central potential in (2+1)-flavor QCD


PACS-CS gauge config. (Clover + Iwasaki)
a = 0.09 fm, L = 2.9 fm
m_π = 700 MeV

⇒ Physical point simulations (m_π = 135 MeV, L = 9.6 fm) at KEI computer
BB forces from LQCD

- Physical origin of the short distance NN repulsion?
- Fate of the H-dibaryon?
- Effect of the SU(3) breaking?

\[ 8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a \]

Symmetric \quad Anti-symmetric
Lattice BB wave functions (flavor SU(3) limit)

Iwasaki + clover
(CP-PACS/JLQCD config.)
$L = 1.9 \text{ fm}, a = 0.12 \text{ fm}, 16^3 \times 32$
$m_\pi = 835 \text{ MeV}, m_B = 1752 \text{ MeV}$

HAL QCD Coll.

Short range BB int. ⇔ Quark Pauli principle

1 : allowed, 27 : partially blocked, 8s : blocked

(Oka, Yazaki, ..... )
BB potentials (flavor SU(3) limit)

Repulsive core in NN channel

Growing NN tensor force

\[ V^{(27)}(r) \]

\[ V^{(10^8)}(r) \]

\[ V_T^{(10^8)}(r) \]

\[ M_{PS} = 1171 \text{ [MeV]} \]
\[ M_{PS} = 1015 \text{ [MeV]} \]
\[ M_{PS} = 837 \text{ [MeV]} \]
\[ M_{PS} = 672 \text{ [MeV]} \]
\[ M_{PS} = 469 \text{ [MeV]} \]
NN phase shifts in the SU(3) symmetric world

Stronger attraction in the deuteron channel

HAL QCD Coll.,
BB potentials (flavor SU(3) limit)

- Repulsive core in NN channel
- Attractive core in H channel
H-dibaryon (flavor SU(3) limit)

At physical point:

\[ M_{\Lambda\Lambda} < M_H < M_{\Xi N} \]

IHalon QCD Coll.
Just for fun: Neutron star from NN potential in flavor SU(3) limit

EOS with Lattice NN force by BHF calculation $\rightarrow$ M-R relation by TOV equation

Inoue et al. [HAL QCD Coll.] (2012)
Just for fun: Neutron star from NN potential in flavor SU(3) limit

EOS with Lattice NN force by BHF calculation \(\Rightarrow\) M-R relation by TOV equation

Inoue et al. [HAL QCD Coll.] (2012)
SU(3) breaking: coupled channel LQCD

\[
(k_n^2 + \nabla^2) \phi_n^\alpha(\mathbf{r}, t) = \int U(\mathbf{r}, \mathbf{r}')^{\alpha\beta} \phi_n^{\beta}(\mathbf{r}', t) d^3r'
\]

Example: \(S=-1, \ 3S_1, \ I=1/2\) \(\left(m_\pi/m_K = 0.89, 0.8\right)\)

\(\Lambda N-\Lambda N\)
\(\Sigma N-\Sigma N\)
\(\Lambda N-\Sigma N\)

PACS-CS (2+1)-flavor config.
L=2.9 fm

⇒ Parallel V (Oct.4) by K. Sasaki
1. LQCD would replace phenomenological interactions in nuclear physics by 1st principle interactions

2. LQCD results together with nuclear many-body techniques would provide us with a firm basis of nuclear physics from QCD

3. Physical point simulations with a large volume (L=9.6 fm) is started at KEI computer (Kuramashi, Ukita et al.)

4. Necessary equipment to extract physics from LQCD data are ready. (e.g. HAL QCD methodology)
“Lattice Quantum Chromodynamical Approach to Nuclear Physics”
[HAL QCD Collaboration]
Progress of Theoretical and Experimental Physics, (2012) 01A105

- basic concepts of the non-local potential
- central, tensor, LS forces from lattice QCD
- coupled-channel YN, YY forces
- three-body force
- kaon-nucleon interaction
- going beyond the pion threshold

➔ Poster (Oct.2) by K. Murano
➔ Talks at Parallel V (Oct.4) by K. Sasaki and Y. Ikeda