Heavy hadron spectroscopy. A quark model perspective.
The November Revolution: 1974

BNL Experimental Observation of a Heavy Particle $J^+$

$M = 3.1 \text{ GeV}$
$\Gamma \sim 0 \text{ MeV}$

SLAC Discovery of a Second Narrow Resonance in $e^+e^-$ Annihilation*

$M = 3.105 \text{ GeV}$
$\Gamma < 1.3 \text{ MeV}$

$M = 3.695 \text{ GeV}$
$\Gamma = 2.7 \text{ MeV}$
The interpretation of the narrow boson resonances at 3.1 and 3.7 GeV as charmed quark-antiquark bound states implies the existence of other states. Some of these should
A quiet period: 1974-2003

FIG. 7. The charmed mesons (\(-cd, cu, cs\)). The legend is as for Fig. 3. Significant spectroscopic mixing in these sectors:
The beginning of a new era: 2003

Observation of a Narrow Meson State Decaying to $D_s^+ \pi^0$ at a Mass of 2.32 GeV/$c^2$

We have observed a narrow state near 2.32 GeV/$c^2$ in the inclusive $D_s^+ \pi^0$ mass distribution from $e^+e^-$ annihilation data at energies near 10.6 GeV. The observed width is consistent with the experimental resolution. The small intrinsic width and the quantum numbers of the final state indicate that the decay violates isospin conservation. The state has natural spin-parity and the low mass suggests a $D_s^*$ assignment. The data sample corresponds to an integrated luminosity of 91 fb$^{-1}$ recorded by the BABAR detector at the SLAC PEP-II asymmetric-energy $e^+e^-$ storage ring.

$D_s^*(2317)^+$, $J^P=0^+$, $\Gamma<3.8$ MeV

$D_s^*(2460)$, $J^P=1^+$, $\Gamma<3.5$ MeV

This mass value and the absence of a strong signal in the $\Upsilon(3S)$ decay channel are in some disagreement with potential model expectations for the $D_s^*$ charmonium state. The mass is within errors at the $D^0D_s^0$ mass threshold (3871.1 ± 1.0 MeV [9]), which is suggestive of a loosely bound $DD_s$ multiquark “molecular state.”

$X(3872)$
$\Gamma<2.3$ MeV

$D_s^*(2112)^+\gamma$, or $D_s^* \gamma\gamma$. Since a $c\bar{s}$ meson of this mass contradicts current models of charm meson spectroscopy [6–8], either these models need modification or the observed state is of a different type altogether, such as a four-quark state.
Etc...

May you live in interesting times

How to proceed?
\[ |\text{meson}\rangle = \alpha_1 |q\bar{q}\rangle + \alpha_2 |qq\bar{q}\rangle + \alpha_3 |qq\bar{q}q\rangle + \ldots \]

\[ |\text{baryon}\rangle = \alpha_1 |qqq\rangle + \alpha_2 |qqq\bar{q}\rangle + \alpha_3 |qqq\bar{q}q\rangle + \ldots \]

Naive Quark Model

\[ 3 \otimes 3 = 1 \]

\[ 3 \otimes \bar{3} = 1 \]

\[ 3 \otimes 3 \otimes \bar{3} \rightarrow \begin{cases} 3_{qq} \otimes 3_{q\bar{q}} = 1 \\ 6_{qq} \otimes 6_{q\bar{q}} = 1 \end{cases} \]

\[ 3 \otimes 3 \otimes 3 \otimes 3 \rightarrow \begin{cases} 1_{qqq} \otimes 1_{q\bar{q}} = 1 \\ 8_{qq} \otimes 8_{q\bar{q}} = 1 \end{cases} \]

\[ 3 \otimes 3 \otimes 3 \otimes \bar{3} \rightarrow \begin{cases} 3_{qq} \otimes 3_{q\bar{q}} \otimes \bar{3}_{q\bar{q}} = 1 \\ 3_{qq} \otimes 3_{q\bar{q}} \otimes 3_{q\bar{q}} = 1 \end{cases} \]
Color 3, 6 = C-parity is a good symmetry

\[ |\Psi\rangle = |\text{Color}\rangle |\text{Isospin}\rangle [|\text{Spin}\rangle \otimes |R\rangle]^{JM} \]

Identical quarks → Pauli

C-parity is a good symmetry

Solving the Schrödinger equation for a 4q system: VM and HH
\[ [(q_1 q_2)(\bar{q}_3 \bar{q}_4)] = \{ [3_{12} 3_{34}], [6_{12} 6_{34}] \} = \{ [\bar{3}_{12}]^{12}_c, [\bar{6}_{12}]^{12}_c \} \\
[(q_1 \bar{q}_3)(q_2 \bar{q}_4)] = \{ [1_{13} 1_{24}], [8_{13} 8_{24}] \} = \{ [11]_c, [88]_c \} \\
[(q_1 \bar{q}_4)(q_2 \bar{q}_3)] = \{ [1_{14} 1_{23}], [8_{14} 8_{23}] \} = \{ [11']_c, [8'8']_c \} \]

\[ P = |11\rangle_c \langle 11| \quad \hat{P} = |1'1'\rangle_c \langle 1'1'| \]
\[ Q = |88\rangle_c \langle 88| \quad \hat{Q} = |8'8'\rangle_c \langle 8'8'| \]
\[ P + Q = I \quad \hat{P} + \hat{Q} = I \]

\[ |\Psi\rangle = \frac{1}{2} \left( \hat{P} \hat{Q} + \hat{Q} \hat{P} \right) \frac{1}{1 - \cos^2 \alpha} |\Psi\rangle + \frac{1}{2} \left( \hat{P} \hat{Q} + \hat{Q} \hat{P} \right) \frac{1}{1 - \cos^2 \alpha} |\Psi\rangle \]

\[ |\Psi\rangle = \mathcal{D} |11\rangle_c |\Psi\rangle + \mathcal{D} |1'1'\rangle_c |\Psi\rangle \]

\[ 11\rangle_c = \cos \alpha |1'1'\rangle_c + \sin \alpha |8'8'\rangle_c \]

\[ 88\rangle_c = \sin \alpha |1'1'\rangle_c + \cos \alpha |8'8'\rangle_c \]

Physical channels

Mesones $D_{sJ}$

$D^*_s(2317) = \alpha_1 |c\bar{s}\rangle + \beta_1 |c\bar{n}\bar{n}\rangle$

$D_s(2460) = \alpha_2 |c\bar{s}\rangle + \beta_2 |c\bar{n}\bar{n}\rangle$

$D^*_s(2308) = \alpha_3 |c\bar{n}\rangle + \beta_3 |c\bar{n}\bar{n}\rangle$

<table>
<thead>
<tr>
<th>Transitions / $D_{sJ}^* \rightarrow D\pi$</th>
<th>QM</th>
<th>[1]</th>
<th>CLEO</th>
<th>Belle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{sJ}^<em>(2317) \rightarrow D_s^</em>\gamma$</td>
<td>0.16</td>
<td>0.17</td>
<td>&lt; 0.059</td>
<td>&lt; 0.18</td>
</tr>
<tr>
<td>$D_{sJ}^*(2317) \rightarrow D_s\gamma$</td>
<td>0</td>
<td>0</td>
<td>&lt; 0.052</td>
<td>&lt; 0.05</td>
</tr>
<tr>
<td>$D_{sJ}(2460) \rightarrow D_s^*\gamma$</td>
<td>0.006</td>
<td>0.47</td>
<td>&lt; 0.16</td>
<td>&lt; 0.31</td>
</tr>
<tr>
<td>$D_{sJ}(2460) \rightarrow D_s\gamma$</td>
<td>0.67</td>
<td>0.51</td>
<td>&lt; 0.49</td>
<td>0.55±0.15</td>
</tr>
</tbody>
</table>


$\Gamma[D_{sJ}(2460) \rightarrow D_s\gamma] / \Gamma[D_{sJ}(2460) \rightarrow D_s^*\gamma] = 100 |\varphi\bar{\varphi} + \psi\bar{\psi}\rangle$

$\Gamma[D_{sJ}(2460) \rightarrow D_s\gamma] / \Gamma[D_{sJ}(2460) \rightarrow D_s^*\gamma] = 1 |\varphi\bar{\varphi}\rangle$
System: $c\bar{c}n\bar{n}$. Model: BCN

There are no non-exotic deeply four-quark bound states (compact)
The role of the thresholds.

Theoretical Thresholds

Experimental Thresholds

Pay attention to your thresholds!!!
The gift from nature to hadronic physicists

\[ cc\bar{n}\bar{n} \text{ and } bb\bar{n}\bar{n} \]

PHYSICAL REVIEW D  VOLUME 25, NUMBER 9  1 MAY 1982

Do narrow heavy multiquark states exist?

J.-P. Ader  J.-M. Richard  P. Taxil

(Received 11 August 1981)

We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. [...] 

These states cannot camouflage themselves in the mesonic jungle

Our qualitative conclusions are certainly rather general. The cryptoexotic configuration \(QQ'\bar{Q}'\), lies above its lowest dissociation threshold \(Q\bar{Q} + Q'\bar{Q}'\). On the other hand, the genuine exotic \(QQQ'\bar{Q}'\) can be stable against dissociation if the ratio of the quark masses is large enough. Our predictions...
System: $c\bar{c}c\bar{c}$. Model: CQC

$4q$ energies

$M_1M_2$ threshold

One compact state in the $cc\bar{c}\bar{c}$ system ($I^p=1^+$)
✓ No compact bound states in the \( c\bar{c}n\bar{n} \) and \( b\bar{b}n\bar{n} \) sectors.

✓ One compact bound state in the \( cc\bar{c}n \) sector and four/three in the \( bb\bar{b}n \) sector.
What about the existence of slightly bound states very close to the threshold?
We solved the scattering of two-meson systems in a coupled-channel approach by means of the Lippmann-Schwinger equation, looking for attractive channels.

We study the consequences of allowing for the reordering of quarks (I) or not (II).
There are no charged partners of the X(3872) [diquark-antidiquark]
Formalisms based on meson-meson and four-quark configurations are fully compatible if they incorporate all the relevant basis vectors (channels)!
There should not be a partner of the $X(3872)$ in the bottom sector.

There should be a $J^P=1^+$ bound state in the exotic bottom sector.
Identifying Multiquark Hadrons from Heavy Ion Collisions

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3RIKEN Nishina Center, Higashi, Wako, Saitama 351-0198, Japan
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5Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843, USA
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(Received 10 November 2010; published 24 May 2011)

Identifying hadronic molecular states and/or hadrons with multiquark components either with or without exotic quantum numbers is a long-standing challenge in hadronic physics. We suggest that studying the production of these hadrons in relativistic heavy ion collisions offers a promising resolution to this problem as yields of exotic hadrons are expected to be strongly affected by their structures. Using the coalescence model for hadron production, we find that, compared to the case of a nonexotic hadron with normal quark numbers, the yield of an exotic hadron is typically an order of magnitude smaller when it is a tetraquark state and a factor of 2 or more larger when it is a molecular state. We further find that some of the newly proposed heavy exotic states could be produced and realistically measured in these experiments.
1985 Bjorken: “We should strive to study triply charmed baryons because their excitation spectrum should be close to the perturbative QCD regime. For their size scales the quark-gluon coupling constant is small and therefore the leading term in the perturbative expansion may be enough.”

- The larger the number of heavy quarks the simpler the system
- nQQ and QQQ $\Rightarrow$ one-gluon exchange and confinement
- nnQ $\Rightarrow$ there is still residual interaction between light quarks
- nnQ and nQQ $\Rightarrow$ the presence of light and heavy quarks may allow to learn about the dynamics of the light diquark subsystem
- Ideal systems to check the assumed flavor independence of confinement
<table>
<thead>
<tr>
<th>State</th>
<th>( J^p )</th>
<th>( Q=\text{Strange} )</th>
<th>( Q=\text{Charm} )</th>
<th>( Q=\text{Bottom} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda \ (udQ) )</td>
<td>1/2(^+)</td>
<td>1116, 1600</td>
<td>2280, 2765</td>
<td>5619</td>
</tr>
<tr>
<td></td>
<td>3/2(^+)</td>
<td>1890</td>
<td>2940(^3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1/2(^-)</td>
<td>1405, 1670</td>
<td>2595</td>
<td>5912(^8)</td>
</tr>
<tr>
<td></td>
<td>3/2(^-)</td>
<td>1520, 1690</td>
<td>2628, 2880(^1)</td>
<td>5920(^8)</td>
</tr>
<tr>
<td></td>
<td>5/2(^+)</td>
<td>1820</td>
<td>2880(^1)</td>
<td></td>
</tr>
<tr>
<td>( \Sigma \ (uuQ) )</td>
<td>1/2(^+)</td>
<td>1193, 1660</td>
<td>2454</td>
<td>5811(^5)</td>
</tr>
<tr>
<td></td>
<td>3/2(^+)</td>
<td>1385, 1840</td>
<td>2518, 2940(^3)</td>
<td>5833(^5)</td>
</tr>
<tr>
<td></td>
<td>1/2(^-)</td>
<td>1480, 1620</td>
<td>2765(^1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3/2(^-)</td>
<td>1560, 1670</td>
<td>2800(^2)</td>
<td></td>
</tr>
<tr>
<td>( \Xi \ (usQ) )</td>
<td>1/2(^+)</td>
<td>1318</td>
<td>2471, 2578</td>
<td>5792(^5,6)</td>
</tr>
<tr>
<td></td>
<td>3/2(^+)</td>
<td>1530</td>
<td>2646, 3076(^2)</td>
<td>5945(^7)</td>
</tr>
<tr>
<td></td>
<td>1/2(^-)</td>
<td></td>
<td>2792, 2980(^3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3/2(^-)</td>
<td>1820</td>
<td>2815</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5/2(^+)</td>
<td></td>
<td>3055(^3), 3123(^3)</td>
<td></td>
</tr>
<tr>
<td>( \Omega \ (ssQ) )</td>
<td>1/2(^+)</td>
<td></td>
<td>2698</td>
<td>6054(^5) (6165(^6))</td>
</tr>
<tr>
<td></td>
<td>3/2(^+)</td>
<td>1672</td>
<td>2768(^3)</td>
<td></td>
</tr>
<tr>
<td>( \Xi \ (uQQ) )</td>
<td>3/2(^-)</td>
<td>--</td>
<td>3519(^4)</td>
<td></td>
</tr>
</tbody>
</table>

**PDG (1990-1995)**

**arXiv:1205.3452**


**Regularities**

\[ \Delta L = 1 \quad 300 \text{ MeV} \]

\[ \Delta n = 1 \quad 500 \text{ MeV} \]
### Heavy baryons: Spin splitting

<table>
<thead>
<tr>
<th>$\Delta M$ (MeV)</th>
<th>Exp.</th>
<th>OGE(μ) $^\text{[1]}$</th>
<th>OGE + OPE $^\text{[2]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_c(3/2^+)$ – $\Lambda_c(1/2^+)$</td>
<td>232</td>
<td>251</td>
<td>217</td>
</tr>
<tr>
<td>$\Sigma_c(3/2^+)$ – $\Sigma_c(1/2^+)$</td>
<td>64</td>
<td>64</td>
<td>67</td>
</tr>
<tr>
<td>$\Sigma_b(3/2^-)$ – $\Lambda_b(1/2^+)$</td>
<td>209</td>
<td>246</td>
<td>205</td>
</tr>
<tr>
<td>$\Sigma_b(3/2^-)$ – $\Sigma_b(1/2^+)$</td>
<td>22</td>
<td>25</td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\Sigma(1/2^+)$</th>
<th>OGE+OPE</th>
<th>OPE=0</th>
<th>$\Delta E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_b(1/2^+)$</td>
<td>5807</td>
<td>5822</td>
<td>– 15</td>
</tr>
<tr>
<td>$\Sigma_b(3/2^+)$</td>
<td>5829</td>
<td>5844</td>
<td>– 15</td>
</tr>
<tr>
<td>$\Lambda_b(1/2^+)$</td>
<td>5624</td>
<td>5819</td>
<td>– 195</td>
</tr>
<tr>
<td>$\Lambda_b(3/2^+)$</td>
<td>6388</td>
<td>6387</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>$\Sigma(1/2^+)$</td>
<td>1408</td>
<td>1417</td>
<td>– 9</td>
</tr>
<tr>
<td>$\Sigma(3/2^+)$</td>
<td>1454</td>
<td>1462</td>
<td>– 8</td>
</tr>
<tr>
<td>$\Lambda(1/2^+)$</td>
<td>1225</td>
<td>1405</td>
<td>– 180</td>
</tr>
</tbody>
</table>

**Doubly charm baryons ⇒ no OPE**

<table>
<thead>
<tr>
<th></th>
<th>$\Xi_{cc}(1/2^+)$</th>
<th>$\Omega_{cc}(1/2^+)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGE + OPE</td>
<td>3579</td>
<td>3697 (118)</td>
</tr>
<tr>
<td>OGE(μ)</td>
<td>3676</td>
<td>3815 (139)</td>
</tr>
<tr>
<td>Lattice</td>
<td>3588</td>
<td>3698 (110)</td>
</tr>
</tbody>
</table>

Summary

• Constituent quark models are an **important tool** to study heavy hadron spectroscopy, however, like all powerful tools have to be handled carefully.

• **Hidden flavor components**, *unquenching the quark model beyond the “naive” approximation*, seem to be neccessary to tame the bewildering landscape of hadrons, but an amazing folklore is borning around.

• **Simultaneous** study of $nnQ$ and $nQQ$ baryons has to be a priority to understand low-energy QCD.

• **Compact** four-quark bound states with **non-exotic** quantum numbers are **hard** to justify while “many-body (medium)” effects do not enter the game.

• **Exotic** many-quark systems should **exist** if our understanding of the dynamics does not hide some information. I hope experimentalists can answer this question to help in the advance of hadron spectroscopy.
Acknowledgements

Let me thank the people I collaborated with in the different subjects I covered in this talk:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Valcarce (Univ. Salamanca, Spain)</td>
<td>J.M. Richard (Grenoble, France)</td>
</tr>
<tr>
<td>T. F. Caramés (Univ. Salamanca, Spain)</td>
<td>F. Fernández (Univ. Salamanca, Spain)</td>
</tr>
<tr>
<td>N. Barnea (Hebrew Univ. Jerusalem, Israel)</td>
<td>H. Garcilazo (IPN, Méjico)</td>
</tr>
<tr>
<td>E. Weissman (Hebrew Univ. Jerusalem, Israel)</td>
<td>B. Silvestre-Brac (Grenoble, France)</td>
</tr>
</tbody>
</table>

Thanks!
The four-quark zoo. What to expect.

Unbound states
The four-quark zoo. What to expect.

Molecular states

Compact states
Meson-Meson molecules Vs. Four-quark bound states (I).

- Figures of merit.

\[ \Delta_E = E_{4q} - E(M_1, M_2) \]

\[ \Delta_R = \frac{RMS_{4q}}{RMS_{M_1} + RMS_{M_2}} \]

\( \Delta_E \geq 0 \rightarrow \text{Unbound state} \)
\( \Delta_E < 0 \rightarrow \text{Bound state} \)

\( \Delta_R \rightarrow \infty \Rightarrow \text{Unbound state} \)
\( \Delta_R \approx 1 - 2 / < 1 \Rightarrow \text{Bound state} \)

- Physical channel: A vector belonging to the Hilbert space whose quantum numbers allow to identify it with two physical mesons.

\[ |(c\bar{c}, c_{13} = 1, s_{13} = 0, i_{13} = 0) (q\bar{q}, c_{24} = 1, s_{24} = 1, i_{24} = 0)\rangle \]
\( (S = 1, I = 0, L = 0, P = +, C = -) \)
\[ \rightarrow |\eta_c \omega\rangle \]
\( \text{Meson-Meson molecules Vs. Four-quark bound states (II).} \)

- **Unbound state (threshold state).** An state with \( \Delta_E \geq 0, \Delta_R \rightarrow \infty \), and whose probability ends into a unique physical channel.

- **Meson-Meson molecule.** An state with \( \Delta_E < 0, \Delta_R \) finite \( \sim 1-2 \), and whose probability ends dominantly into a single physical channel.

- **Compact four-quark state.** An state with \( \Delta_E < 0, \Delta_R < 1 \), and whose wave function contains several different physical channels.
Probability of physical channels vs. Binding energy

| $(S_T, I)$ | (0,1) |
| Flavor     | $cc\bar{n}\bar{n}$ |
| Energy     | 3877 |
| Threshold  | $DD |_{S}$ |
| $\Delta E$ | +5 |

We multiply the interaction between the light quarks by a fudge factor. This modifies the 4q energy but not the threshold.
We have analyzed all positive parity channels until $J^P = 2^+$. Only one channel is attractive:

$$(I) J^P = (0) 1^+$$

Explicit flavor sector: Exotics

Formalisms based on meson-meson configurations and those considering explicitly four-quark states are equivalent if (and only if) a full basis is considered.
Meson-baryon threshold effects in the light-quark baryon spectrum

P. Gonzalez et al.

We argue that selected S wave meson-baryon channels may play a key role in matching poor baryon mass predictions from quark models with data. The identification of these channels with effective inelastic channels in data analysis allows us to derive a prescription that could improve the extraction and identification of baryon resonances.
Hidden and open-charm meson spectra

\[ V_{\text{II}}^{(3)} = \frac{1}{2} \sum_{i \neq j \neq k \neq i} \frac{V_0}{m_i m_j m_k} \frac{e^{-m_0 r_{ij}} e^{-m_0 r_{ik}}}{m_0 r_{ij}} \]

\[ V_{\text{III}}^{(3)} = V_0 \exp \left(-\sum_{i<j} \frac{r_{ij}^2}{\lambda^2} \right) \]
\[ R(r_{\alpha}, r_{\beta}) = r_{\alpha} r_{\beta} \sum \int_{\mathcal{V}} \left| R_i(\vec{r}_\alpha, \vec{r}_\beta, \vec{r}_\gamma) \right|^2 d\vec{r}_\gamma d\Omega_{r_{\alpha}} d\Omega_{r_{\beta}} \]
Beyond two-body interactions.

\( \Delta E = -uE(M_1, M_2) \)

\((I) \to \text{Flip-flop}: V_I = \lambda \min(r_{i_3} + r_{24}, r_{23} + r_{i_4}) \)

\((II) \to \text{Butterfly}: V_{II} = \lambda \min(r_{1k} + r_{2k} + r_{kl} + r_{i_3} + r_{i_4}) \)

\[ V_{4q} = \min(V_I, V_{II}) \]

\( (p, p, e^-, e^-) \) more stable than \( (e^+, e^-, e^+, e^-) \)

\( (M^+, M^-, m^+, m^-) \) becomes unstable for \( M / m > 2.2 \)
Capabilities of the HH and VM methods.

**L=0 S=1 I=0 cc\(\bar{c}\)n\(\bar{n}\)**

<table>
<thead>
<tr>
<th>Method</th>
<th>HOD*</th>
<th>SVA*</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3931.0</td>
<td>3904.7</td>
<td>3899.2</td>
</tr>
</tbody>
</table>

**SVA*:** Stochastic variational approach (BCN).

**HOD*:** Diagonalization in a harmonic oscillator basis up to N=8 (BCN).

**L=0 cc\(\bar{c}\)n\(\bar{n}\) states**

<table>
<thead>
<tr>
<th>(S,I)</th>
<th>VMCT*</th>
<th>HH ((\ell_i=0))</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>4155</td>
<td>4154</td>
<td>3911</td>
</tr>
<tr>
<td>(1,0)</td>
<td>3927</td>
<td>3926</td>
<td>3860</td>
</tr>
<tr>
<td>(1,1)</td>
<td>4176</td>
<td>4175</td>
<td>3975</td>
</tr>
<tr>
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<td>4031</td>
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**HH**

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<th>RMS</th>
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<tr>
<td>3860.6</td>
<td>0.367</td>
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**VM**

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<td>3861.4</td>
<td>0.363</td>
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**VMCT*:** Variational calculation using gaussian trial wave functions with only quadratic terms in the Jacobi coordinates (CQC).
FIG. 1: Experimental masses of the different two meson systems made of a heavy and a light quark and their corresponding antibaryons, $QnQ\bar{n}$ with $Q = s, c, or b$, for several sets of quantum numbers, $J^{PC}$. We have set as our origin of energies the $KK$, $DD$ and $BB$ masses for the hidden strange, charm and bottom sectors, respectively.