Symmetry energy: From finite nuclei to neutron stars

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and
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Asymmetry

Atomic Nucleus

Proton

Neutron
Asymmetry

Atomic Nucleus

N ≟ Z often N > Z

Microscopically, ρ_n ≠ ρ_p

ρ_n > ρ_p
Asymmetry is coming

Atomic Nucleus

N ≠ Z often N > Z

Microscopically, \( \rho_n \neq \rho_p \)

\[ \rho_n > \rho_p \]
Asymmetry in nuclear systems
Finite Nuclei

- Coulomb energy $\propto \frac{Z^2}{A^{1/3}}$
- Symmetry energy $\propto \frac{(N-Z)^2}{A}$
Asymmetry in nuclear systems

Finite Nuclei

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Neutron Star

- $\beta$ equilibration
- Charge neutrality
Asymmetry in nuclear systems

Finite Nuclei
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- Symmetry energy $\propto \frac{(N-Z)^2}{A}$

Neutron Star
- $\beta$ equilibration
- Charge neutrality
  Nucleus $\rightarrow 0.16$ fm$^{-3}$
  Core of NS $\rightarrow \sim 0.8$ fm$^{-3}$

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Symmetric Nuclear Matter

\(\rho_n = \rho_p\)
Symmetric Nuclear Matter

\( (\rho_n = \rho_p) \)

\[ \epsilon_0 = \frac{E}{A}(\rho_0), \]

\[ K_0 = 9\rho_0^2 \left( \frac{\partial^2 (E/A)_{SNM}}{\partial \rho^2} \right)_{\rho_0}. \]
Nuclear Matter Properties: (SNM) and (PNM)

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Pure Neutron Matter
\((\rho = \rho_n; \rho_p = 0)\)
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\[E_{\text{sym}}(\rho) = \left(\frac{E}{A}\right)_{PNM}(\rho) - \left(\frac{E}{A}\right)_{SNM}(\rho).\]
Nuclear Matter Properties: (SNM) and (PNM)

**Symmetric Nuclear Matter**

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**Pure Neutron Matter**

\( \rho = \rho_n; \rho_p = 0 \)

\[ E_{sym}(\rho) = (E/A)_{PNM}(\rho) - (E/A)_{SNM}(\rho). \]

\[ J = E_{sym}(\rho_0), \]

\[ L = \left( 3\rho \frac{\partial E_{sym}}{\partial \rho} \right)_{\rho_0}. \]
• Binding Energy $\rightarrow \epsilon_0 \, (\sim -16 \text{ MeV})$
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Nuclear Matter Properties
Constraints from Experiments

- Binding Energy \( \rightarrow \epsilon_0 (\sim -16 \text{ MeV}) \)
- Charge radii \( \rightarrow \rho_0 (\sim 0.16 \text{ fm}^{-3}) \)
- Monopole resonance \( \rightarrow K_0 (\sim 230 \text{ MeV}) \)
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- Binding energy of asymmetric nuclei $\rightarrow J \ (\sim 32 \text{ MeV})$
- $L \leftarrow \text{????}$
Slope of symmetry energy (L)
Connection with experimental quantities

\[
\Delta r_{np} = \left\langle r_n^2 \right\rangle^{\frac{1}{2}} - \left\langle r_p^2 \right\rangle^{\frac{1}{2}}.
\]

Slope of symmetry energy \((L)\)

Connection with experimental quantities

\[
\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}.
\]

Measurement:

\[
\Delta r_{np} = 0.33^{+0.16}_{-0.18} \text{ fm}
\]

\[
L = 35-135 \text{ MeV}
\]

S. Abrahamyan et al.,


$\Delta r_{np}$ and L parameter

From binding energies of nuclei

$\Delta B (^{68}\text{Ni}, ^{56}\text{Ni})$ (MeV)

$\Delta B (^{24}\text{O}, ^{16}\text{O})$ (MeV)

$\Delta r_{np} (^{208}\text{Pb})$ (fm)

$\Delta B (^{132}\text{Sn}, ^{100}\text{Sn})$ (MeV)

$\Delta B (^{30}\text{Ne}, ^{18}\text{Ne})$ (MeV)

$\Delta r_{np} (^{208}\text{Pb})$ (fm)

$r = -0.586$

$r = 0.012$

$r = 0.980$

$r = 0.978$

$\frac{N}{Z} (^{68}\text{Ni})=1.43,$

$\frac{N}{Z} (^{132}\text{Sn})=1.64,$

$\frac{N}{Z} (^{30}\text{Ne}, ^{24}\text{O})=2.0$

CM, B. K. Agrawal et. al., PRC 92, 024302 (2015)
\( \Delta r_{np} \) and L parameter

From binding energies of nuclei

<table>
<thead>
<tr>
<th>( \Delta B(\text{Ni}) ) (MeV)</th>
<th>( \Delta B(\text{O}) ) (MeV)</th>
</tr>
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<tbody>
<tr>
<td>(68 Ni, 56 Ni)</td>
<td>(24 O, 16 O)</td>
</tr>
<tr>
<td>(132 Sn, 100 Sn)</td>
<td>(30 Ne, 18 Ne)</td>
</tr>
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\[ r = -0.586 \quad r = 0.012 \quad r = 0.980 \quad r = 0.978 \]

\( \frac{N}{Z}(68 \text{Ni}) = 1.43, \quad \frac{N}{Z}(132 \text{Sn}) = 1.64, \quad \frac{N}{Z}(30 \text{Ne}, 24 \text{O}) = 2.0 \]

Fitting protocol

SINPA ⇒ Standard set- \( ^{16}\text{O}, ^{40,48,54}\text{Ca}, ^{56,68,78}\text{Ni}, ^{90}\text{Zr}, \)
\( ^{100,132,138}\text{Sn}, ^{144}\text{Sm}, ^{208}\text{Pb}, \)
\( ^{24}\text{O}, ^{30}\text{Ne}, ^{36}\text{Mg}, ^{58}\text{Ca} \) and \( M_{\text{NS}}^{\text{max}} \)

CM, B. K. Agrawal et al., PRC 92, 024302 (2015)
$\Delta r_{np}$ and $L$ parameter
From binding energies of nuclei

Fitting protocol

SINPA $\Rightarrow$ Standard set- $^{16}\text{O}$, $^{40,48,54}\text{Ca}$, $^{56,68,78}\text{Ni}$, $^{90}\text{Zr}$, $^{100,132,138}\text{Sn}$, $^{144}\text{Sm}$, $^{208}\text{Pb}$, $^{24}\text{O}$, $^{30}\text{Ne}$, $^{36}\text{Mg}$, $^{58}\text{Ca}$ and $M^{\text{NS}}_{\text{max}}$

$\frac{N}{Z}(^{24}\text{O}, ^{30}\text{Ne}, ^{36}\text{Mg}, ^{58}\text{Ca}) \sim 2.0$

$L = 54 \pm 5\text{MeV}$

$\frac{N}{Z}(^{68}\text{Ni})=1.43$, $\frac{N}{Z}(^{132}\text{Sn})=1.64$, $\frac{N}{Z}(^{30}\text{Ne}, ^{24}\text{O})=2.0$

CM, B. K. Agrawal et. al., PRC 92, 024302 (2015)
Sensitivity analysis in SINPA
Importance of Nuclei with $\frac{N}{Z} \sim 2$
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Importance of Nuclei with $\frac{N}{Z} \sim 2$

\[ S(p) = [J(p)J^T(p)]^{-1}J(p) \]

with $J_{\alpha i} = \frac{1}{\Delta O_i} \left( \frac{\partial O_i}{\partial p_\alpha} \right)_{p_0}$.

\[ S_{\alpha i} \sum_i S_{\alpha i} \times 100 \]

CM, B. K. Agrawal et. al., PRC 93, 044328 (2016).
Sensitivity analysis in SINPA
Importance of Nuclei with $\frac{N}{Z} \sim 2$

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$i^{th}$ datum $O_i$ to $\alpha$-th parameter $p_\alpha$,

$$\frac{S_{\alpha i}}{\sum_i S_{\alpha i}} \times 100$$

Nuclei with $\frac{N}{Z} \sim 2$

$B(^{24}\text{O}, \, ^{30}\text{Ne}, \, ^{36}\text{Mg and \, ^{58}\text{Ca}})$

CM, B. K. Agrawal et. al., PRC 93, 044328 (2016).
Mass-Radius of Neutron Star
Connection to L parameter
Mass-Radius of Neutron Star

Connection to $L$ parameter

Constraints on Maximum mass of NS:
J. Antoniadis et. al., Science 340 (2013) 6131
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Mass-Radius of Neutron Star
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Constraints on Maximum mass of NS:
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Symmetry ..... neutron stars
Nuclear systems are in general asymmetric. Nucleus $\rightarrow \rho_0$.
Core of NS $\rightarrow \sim 4-5 \rho_0$. 
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Symmetry energy and specially its density dependence is thus extremely important in the context of nuclear systems over a wide range of density.
Conclusions

- Nuclear systems are in general asymmetric. Nucleus $\rightarrow \rho_0$. Core of NS $\rightarrow \sim 4-5 \rho_0$.

- Symmetry energy and specially its density dependence is thus extremely important in the context of nuclear systems over a wide range of density.

- If it is determined accurately from laboratory experiments $[B(N/Z \sim 2)]$, predictions can be made for astrophysical observations, e.g. Mass and Radius of compact objects like neutron stars.
Conclusions

- Nuclear systems are in general asymmetric. Nucleus $\rightarrow \rho_0$. Core of NS $\rightarrow \sim 4-5 \rho_0$.

- Symmetry energy and specially its density dependence is thus extremely important in the context of nuclear systems over a wide range of density.

- If it is determined accurately from laboratory experiments [$B(N/Z\sim 2)$], predictions can be made for astrophysical observations, e.g. Mass and Radius of compact objects like neutron stars.

- Future observations promise to measure mass and radius of a neutron star more precisely. This can help us to understand the density dependence of symmetry energy and in turn the basic nature of nucleon-nucleon interaction in medium.
Collaborators

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- S. K. Samaddar (SINP, Kolkata)
- M. Centelles (ICCUB, Spain)
- X. Viñas (ICCUB, Spain)
- C. Gonzalez-Boquera (ICCUB, Spain)
- L. M Robledo (Universidad Autónoma de Madrid, Spain)
“The universe is asymmetric and I am persuaded that life, as it is known to us, is a direct result of the asymmetry of the universe or of its indirect consequences.”

— Louis Pasteur
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Thank You....
\[ \mathcal{L}_{\text{tot}} = \mathcal{L}_b + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_{\tilde{\rho}} + \mathcal{L}_{\sigma \omega \tilde{\rho}}. \]

\[ \mathcal{L}_b = \sum_{J=n,p} \overline{\Psi}_J [i \gamma^\mu \partial_\mu - (M - g_\sigma \sigma) - (g_\omega \gamma^\mu \omega_\mu + \frac{1}{2} g_{\tilde{\rho}} \gamma^\mu \tau. \tilde{\rho}_\mu)] \Psi_J. \]

\[ \mathcal{L}_\sigma = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{\kappa_3}{6M} g_\sigma m_\sigma^2 \sigma^3 - \frac{\kappa_4}{24M^2} g_\sigma^2 m_\sigma^2 \sigma^4 \]

\[ \mathcal{L}_\omega = -\frac{1}{4} \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{24} \zeta_0 g_\omega^2 (\omega_\mu \omega^\mu)^2, \]

\[ \mathcal{L}_{\tilde{\rho}} = -\frac{1}{4} \tilde{\rho}_{\mu \nu} \tilde{\rho}^{\mu \nu} + \frac{1}{2} m_{\tilde{\rho}}^2 \tilde{\rho}_\mu \tilde{\rho}^\mu \]

\[ \mathcal{L}_{\text{mixed}} = \frac{\eta_{\tilde{\rho}}}{2M} g_\sigma m_{\tilde{\rho}}^2 \sigma \tilde{\rho}_\mu \tilde{\rho}^\mu + \frac{\eta_{2\tilde{\rho}}}{4M^2} g_\omega^2 m_{\tilde{\rho}}^2 \omega_\mu \omega^\mu \tilde{\rho}_\nu \tilde{\rho}^\nu \]