Non-Linear Realizations of
Space-Time Symmetries

Joaquim Gomis

Universitat de Barcelona
PH-TH Division, CERN


Firenze. December 2007
Plan of the talk

- Non-Linear Realization of $G$ on $G/H$
- Space-time as a Coset $G/H$
- Massive Relativistic Particles Actions from a Non-Linear Realizations
- Massless Particles
- Relativistic Branes
- Conclusions and Open Problems
Let us consider a super Lie group \( G \) and a subgroup \( H \). The Lie generators are

\[
G_A \in \text{Lie } G, \quad G_i \in \text{Lie } H, \quad G_I \in \text{Lie } G/H
\]  
(1)

They satisfy the graded commutation relations

\[
[G_A, G_B] = if^{C}_{AB}G_C,
\]  
(2)

Consider the coset \( G/H \). Locally we parametrize the coset elements as

\[
g(z) = e^{iG_Iz^I}
\]  
(3)

The Maurer-Cartan 1-form (MC)

\[
\Omega = -ig^{-1}dg \equiv G_AL^A
\]  
(4)

This equation defines the set of 1-forms \( L^A \).

The (MC) equation

\[
d\Omega + i\Omega \wedge \Omega = 0
\]  
(5)
implies the structure equations for the $L^A$'s

$$dL^A + \frac{1}{2} f^A_{BC} L^C \wedge L^B = 0$$  \hspace{1cm} (6)

When the forms $L^A$ are considered as depending on the coset parameters $z^I$, we have

$$L^A = dz^I L^A_I$$  \hspace{1cm} (7)

- **Realization of $G$ on $G/H$**

Consider the left multiplication on a coset element $g(z)$.

$$g(z) \rightarrow g_0 g(z),$$  \hspace{1cm} (8)

it can expressed again in terms of a coset group element times an element of the subgroup $H$, that is

$$g_0 g(z) = g(z') h(z, g_0) \rightarrow g(z') = g_0 g(z) h^{-1}$$  \hspace{1cm} (9)
Therefore we get the transformed MC form

\[ \Omega' = h\Omega h^{-1} - ihdh^{-1}. \] (10)

Since \( h^{-1}dh \) lies in \( \text{Lie } H \) and if \( [H, G/H] = G/H \) the part of \( \Omega \) lying in the coset transforms homogeneously

\[ \Omega'_{G/H} = h\Omega_{G/H}h^{-1} \] (11)

whereas the part lying in \( H \) transforms as a gauge field

\[ \Omega'_{H} = h\Omega_{H}h^{-1} - ihdh^{-1} \] (12)
The space-time is identified with the translations (supertranslations) of an space-time graded Lie algebra

- Minkowski Space $\frac{ISO(d-1,1)}{SO(d-1,1)} = \text{Poincare Lorentz}$

The Poincare algebra is

$$
[M_{mn}, M_{rs}] = -i \eta_{[r} M_{ms]} + i \eta_{m[r} M_{ns]},
$$

$$
[P_m, M_{rs}] = -i \eta_{mr} P_s
$$

$$
[P_m, P_n] = 0.
$$

(13)

where $\eta_{mn} = (-; + + + + \cdots)$

The MC equation is

$$
dL^m + L^\ell L^m_{\ell} = 0,
$$

$$
dL^{mn} + L^{m\ell} L^n_{\ell} = 0,
$$

(14)
The coset is parametrized by

\[ g = e^{iP_mX^m} \quad (15) \]

The explicit form of MC form is

\[ \Omega = -ig^{-1} dg = P_m dX^m, \quad (16) \]

\[ L^m = dX^m, \quad L^{mn} = 0. \quad (17) \]
D-dimensional adS (dS) space is defined as an surface of $D+1$ flat dimensional space

$$\eta_{mn}u^mu^n + \eta^#(u^#)^2 = \eta^# R^2,$$  \hspace{1cm} (18)

for adS $\eta^# = -1$, for dS $\eta^# = +1$. The metric in the ambient space is given by

$$\eta_{mn}du^m du^n + \eta^#(du^#)^2 = ds^2.$$  \hspace{1cm} (19)

The adS (dS) algebra is

$$[P_m, P_n] = \mp \frac{i}{R^2} M_{mn}, \hspace{1cm} \mp \text{ for AdS/S.} \hspace{1cm} (20)$$

$$[P_m, M_{n\ell}] = -i(\eta_{mn}P_\ell - \eta_{m\ell}P_n),$$  \hspace{1cm} (21)

$$[M_{mn}, M_{\ell r}] = -i\eta_{n\ell} M_{mr} + \text{....}$$  \hspace{1cm} (22)

The coset parametrization is

$$g = e^{iX^m P_m} \hspace{1cm} (23)$$

and the explicit form of MC form is

$$\Omega = -ig^{-1} dg = L^m P_m + \frac{1}{2} L^{mn} P_{mn}. \hspace{1cm} (24)$$
where, for adS,

\[ L^m = L^m_n dX^n = [\delta^m_n + \left( \frac{\sin \frac{r}{R}}{\frac{r}{R}} - 1 \right) (\eta^m_n - \frac{X_n X^m}{X^\ell X^\ell})] \] (25)

\[ L^{mn} = \left( \frac{\cos \left( \frac{r}{R} \right) - 1}{X^\ell X^\ell} \right) dX^{[m X^n]} \] (26)

They satisfy the MC equations

\[ dL^m + L^{mn} L_n = 0, \quad dL^{mn} + L^{m \ell} L_\ell^n = -\frac{1}{R^2} L^m L^n \] (27)

Note that \( L^m \) is the vielbein and \( L^{mn} \) is the spin connection.

The metric is

\[ g^{AdS}_m (X) = \eta_{rs} L^r m L^s n = \eta_{mn} + \left( \left( \frac{\sin \frac{r}{R}}{\frac{r}{R}} \right)^2 - 1 \right) (\eta_{mn} - \frac{X_m X^n}{X^\ell X^\ell}) \] (28)

\[ \eta_{mn} = (-; + + + +), \quad r = \sqrt{-X^\ell X^\ell}, \] (29)
Ordinary Superspace = IIASuperPoincare/Lorentz

We have the translations $P_m, Q_\alpha$
The flat susyalgebra is, $m = 0, 1, \ldots, 9, \alpha = 1, \ldots, 32$

\[
\begin{align*}
[M_{mn}, M_{rs}] &= -i \eta_{nr} M_{ms} + \ldots \\
[M_{mn}, P_r] &= -i \eta_{[nr} P_{m]} \\
[P_m, P_r] &= 0 \\
[Q, M_{mn}] &= \frac{i}{2} Q \Gamma_{mn}, \\
[Q, P_m] &= 0, \\
\{Q, Q\} &= 2 C \Gamma^m P_m
\end{align*}
\]

where $C$ is the conjugation matrix

\[
C^T = -C, \quad C^{-1} \Gamma_m^T C = -\Gamma_m, \quad (C \Gamma_m)^T = (C \Gamma_m).
\]

\[
\{\Gamma^m, \Gamma^n\} = +2 \eta^{mn} = +2(-; +\ldots+),
\]

$\Gamma_{11}$ is also real matrix defined by

\[
\Gamma_{11} = \Gamma_0 \Gamma_1 \ldots \Gamma_9,
\]

and satisfies

\[
\{\Gamma^m, \Gamma_{11}\} = 0, \quad \Gamma_{11}^2 = +1
\]
Totally antisymmetric product of $\Gamma$’s is
\[ \Gamma_{mn...r} = \Gamma_m \Gamma_n ... \Gamma_r, \quad \text{(for different indices)} \quad (40) \]

The MC form
\[ \Omega = -i \ g^{-1} \ d \ g = P_m L^m + \frac{1}{2} M_{mn} L^{mn} + Q_{\alpha} L^\alpha \quad (41) \]
satisfies the MC equation
\[ d \Omega + i \Omega \wedge \Omega = 0. \quad (42) \]
that is
\[ dL^m = -L^{mn} L_n + i \bar{L} \Gamma^m L, \]
\[ dL_{mn} = -L_{mr} L_{rn}, \]
\[ dL = -\frac{1}{4} L_{rs} \Gamma_{rs} L. \quad (43) \]

We parametrize the coset as
\[ g = e^{i P_m x^m} \ e^{i Q_\alpha \theta^\alpha}. \quad (44) \]

\[ L^m = dX^m + i \bar{\theta} \Gamma^m d\theta, \ L^\alpha = d\theta^\alpha \quad (45) \]
Chevalley-Eilenberg Cohomology

We can construct an invariant two form that it is closed

$$\Omega_2 = d\bar{\theta}\Gamma_{11}d\theta = d\Omega_1$$  \hspace{1cm} (46)$$

where the 1-form

$$\Omega_1 = \bar{\theta}\Gamma_{11}d\theta$$  \hspace{1cm} (47)$$

is not invariant. The Chevalley-Eilenberg Cohomology at degree two is not trivial. This fact implies the existence of a central extension of the susy algebra

$$\{Q, Q\} = 2C(\Gamma^m P_m + \Gamma_{11}Z).$$  \hspace{1cm} (48)$$

• Vector Superspace

We have the translations $P_\mu, G_\mu, G_5$. The vector susy algebra is

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i\eta_{\nu\rho}M_{\mu\sigma} - i\eta_{\mu\sigma}M_{\nu\rho} + i\eta_{\nu\sigma}M_{\mu\rho} + i\eta_{\mu\rho}M_{\nu\sigma},$$  \hspace{1cm} (49)$$

$$[M_{\mu\nu}, P_\rho] = i\eta_{\mu\rho}P_\nu - i\eta_{\nu\rho}P_\mu, \quad [M_{\mu\nu}, G_\rho] = i\eta_{\mu\rho}G_\nu - i\eta_{\nu\rho}G_\mu,$$  \hspace{1cm} (50)$$

$$[G_\mu, G_\nu]_+ = 0, \quad [G_5, G_5]_+ = 0, \quad [G_\mu, G_5]_+ = -P_\mu.$$  \hspace{1cm} (51)$$

We locally parameterize the coset element as

$$g = e^{iP_\mu x^\mu} e^{iG_5 \xi_5} e^{iG_\mu \xi^\mu}.$$  \hspace{1cm} (52)$$
The Maurer-Cartan form (MC) is

\[ \Omega \equiv G_A L^A = P_\mu dx^\mu + G_5 d\xi^5 + G_\mu d\xi^\mu \] (53)

with

\[ L^\mu_x = dx^\mu - i\xi^\mu d\xi^5, \quad L^\mu_\xi = d\xi^\mu, \quad L^5 = d\xi^5. \] (54)

MC equation=

We can construct two closed invariant 2-forms

\[ \Omega_2 = L^\mu_\xi \wedge L^\nu_\xi \eta_{\mu\nu} = d\xi^\mu \wedge d\xi^\nu \eta_{\mu\nu}, \quad \tilde{\Omega}_2 = d\xi^5 \wedge d\xi^5. \] (55)

These forms are exact, since

\[ \Omega_2 = d\Omega_1 = d(\xi^\mu d\xi^\nu \eta_{\mu\nu}), \quad \tilde{\Omega}_2 = d\tilde{\Omega}_1 = d(\xi^5 d\xi^5). \] (56)

According to Chevalley-Eilenberg cohomology this susy algebra admits two central extensions given by

\[ [G_\mu, G_\nu]_+ = \eta_{\mu\nu} Z, \quad [G_5, G_5]_+ = \tilde{Z}. \] (57)
A massive particle in a space-time background breaks spatial translations and boost symmetry. To describe the action we should consider a new coset

\[ G_{\text{spacetime}} / \text{Rotations} \]  \hspace{1cm} (58)

parametrized by

\[ g = g_L U, \quad U = e^{iM_0i\nu^i} \]  \hspace{1cm} (59)

where \( g_L \) is the coset associated to the space, \( M_0i \) are the Lorentz boost generators and \( \nu^i \) are the corresponding Goldstone parameters. \( U \) represents a finite Lorentz boost. The Maurer-Cartan 1-form is now

\[ \Omega = -ig^{-1}dg = U^{-1}\Omega_LU - iU^{-1}dU \]  \hspace{1cm} (60)

In order to get the explicit expression for the MC form we need to compute the transformation of the Lie algebra generators under finite Lorentz boost. In order to get the action it will be sufficient to consider the transformation of the ordinary translations \( P_m \).

\[ U^{-1}P_mU = P_n\Lambda_{m}^{n}(\nu), \]  \hspace{1cm} (61)
where $\Lambda^m_n(v)$ is a finite Lorentz transformation. The new MC forms associated to the translations are

$$\tilde{L}^n = \Lambda^m_n(v)L^m$$  \hspace{1cm} (62)

The manifest invariant action is

$$S^{NG} = \int (m \tilde{L}^0)^* = \int (m \Lambda^0_m(v)L^m)^* = \int L \, d\tau$$  \hspace{1cm} (63)$$

where $\Lambda^0_\mu$ is a time-like Lorentz vector, $\Lambda^0_\mu \Lambda^0_n \eta^{\mu
u} = -1$. Note that the Goldstone variables $v^i$ are non-dynamical and can be eliminated by their equations of motion. We will not do this here. Instead we will write the previous action in canonical form.

The canonical momenta of $x^\mu$, for the flat cases, are

$$p_m = \frac{\partial L}{\partial \dot{x}^m} = m\Lambda^0_m(v).$$  \hspace{1cm} (64)$$

These momenta verify the mass-shell constraint $p^2 + m^2 = 0$

The Nambu-Goto part of the canonical lagragian can be written as

$$L^{NG} = p \cdot L^* - \frac{e}{2}(p^2 + m^2)$$  \hspace{1cm} (65)$$
For the adS case we have an analogous formula with $p^2$ written in terms of adS metric.

**WZ Terms**

Can we add other terms to the NG piece of the action? Yes, if the Chevalley-Eilenberg cohomology is non-trivial

- **Superparticle**
  
  The 1-form
  
  $$\Omega_1 = \bar{\theta}\Gamma_{11}d\theta$$  \hspace{1cm} (66)
  
  is not invariant but quasi-invariant therefore
  
  $$S^{WZ} = \int \Omega_1^*$$  \hspace{1cm} (67)
  
  is invariant. Therefore the action of the superparticle is
  
  $$S = S^{NG} + q S^{WZ}$$  \hspace{1cm} (68)
  
  where $q$ is call the charge of the particle. In case $m = \pm q$ there exits a fermionic gauge symmetry call kappa symmetry, that can be understood as a local right action on the coset.
• **Spinning particle** There are two 1-forms that are quasi-invariant

\[ \Omega_1 = \xi^\mu d\xi^\nu \eta_{\mu\nu}, \quad \tilde{\Omega}_1 = \xi^5 d\xi^5. \]  

(69)

The WZ action in this case

\[ S^{WZ} = \beta \int \Omega_1^* + \gamma \int \tilde{\Omega}_1^* \]  

(70)

The action for an spinning particle is

\[ S = S^{NG} + S^{WZ} \]  

(71)

Like in the super particle case if \( m^2 = -\beta\gamma \) the cation has a new gauge symmetry, gauge world-line supersymmetry.
Massless Bosonic Particle in 4d

Note that little group for a massless particle is $Euclid_2$

We should consider the coset $\frac{ISO(3,1)}{E(2)} = \frac{Poincare}{Euclid_2}$.

We write the Poincare algebra in terms of the indexes $(+, -, a)$, where $+, -$ are the light cone coordinates with $\eta_{+-} = 1$ and $(a = 2, 3)$.

\[
\begin{align*}
[M_{ab}, M_{cd}] &= -i \, \eta_{[c} \, M_{ad]} + i \, \eta_{a[c} \, M_{bd]}, \\
[M_{\pm a}, M_{cd}] &= -i \, \eta_{a[c} \, M_{\pm d]}, \\
[M_{+ a}, M_{- b}] &= +i \, \eta_{+-} \, M_{ab} + i \, \eta_{ab} \, M_{+-}, \\
[P_{\pm}, M_{\mp a}] &= -i \, P_{a}, \\
[P_{a}, M_{\pm b}] &= +i\eta_{ab} \, P_{\pm}, \\
[P_{a}, M_{bc}] &= -i\eta_{a[b} \, P_{c}].
\end{align*}
\]

The little group for a massless particle has generators $(M_{ab}, M_{-a})$.

Note in this case $[H, G/H] \neq G/H$.

We parametrize the coset as

\[
g = g_0 U. \tag{73}
\]
where \( g_0 = e^{iP_m X^m} \) and \( U = e^{iM+av^a} e^{iM_+ - u} \) is finite lorentz transformation. The MC form is given by

\[
\Omega = P_m L^m + \frac{1}{2} M_{mn} L^{mn}.
\]  

(74)

where

\[
\begin{align*}
L^+ &= e^{-u}(dX^+ - v_a dX^a - \frac{v^2}{2} dX^-), \\
L^- &= e^u dX^-, \\
L^a &= dX^a + v^a dX^-, \\
L^{a+} &= e^{-u} dv^a, \\
L^{+-} &= du, \\
L^{ab} &= 0, L^{-a} = 0.
\end{align*}
\]  

(75)

We can write

\[
L^m = \Lambda^m_n dX^n,
\]  

(76)

where \( \Lambda^m_n \) is finite Lorentz transformation with parameters \( v^a, u \).

\[
\Lambda^{\nu}_{\mu}(v, u) = \begin{pmatrix}
e^{-u} & -e^{-u} v^2 \frac{v^2}{2} & -e^{-u} v_b \\
0 & e^u & 0 \\
0 & v^a & 1
\end{pmatrix}
\]  

(77)
Since $L^+$ is the only invariant one form under $E(2)$ the action with lower order of derivatives is

$$S = \int (\Lambda^+ n(v^a, u) \, dX^n)^*.$$  \hspace{1cm} (78)

Since $\Lambda^+ n$ is a light-like vector we can write the action as

$$S = \int d\tau (\Lambda^+ n \, \dot{X}^n - \frac{e}{2} \eta^{mn} \Lambda^+ m \Lambda^+ n),$$ \hspace{1cm} (79)

Notice that the momenta $p_m = \Lambda^+ m$ we have the ordinary action of a massless bosonic particle.
• **p-Brane in Flat Background**

We now construct the non-linear realisation that leads to dynamics of the bosonic p-brane moving in a $D$-dimensional Minkowski space-time. We consider the coset

$$ISO(1, D - 1)/SO(1, p) \otimes SO(D - p - 1)$$  \hspace{1cm} (80)

The Poincare algebra is

$$[J_{ab}, J_{cd}] = -i\eta_{bc}J_{ad} + i\eta_{ac}J_{bd} + i\eta_{bd}J_{ac} - i\eta_{ad}J_{bc}$$  \hspace{1cm} (81)

$$[J_{ab}, P_c] = -i\eta_{bc}P_a + i\eta_{ac}P_b.$$  \hspace{1cm} (82)

We are using the notation that an underlined index, i.e. $a$ goes over all possible values from 0 to $D - 1$ while the unprimed indices $a$ take the values $a = 0, \ldots, p$ and primed indices $a'$ take the values $p + 1, \ldots, D - 1$. The latter are the indices which are longitudinal and transverse to the brane respectively.

We choose the coset element as

$$g = g_0 \ U, \quad g_0 = e^{i\mathbf{x}_a P_a}, \quad U = e^{i\phi_{a'} b' J^{a'}_{a'}}.$$  \hspace{1cm} (83)
The Cartan forms are defined by
\[ \mathcal{V} = -ig^{-1}dg = U^{-1}(-ig_0^{-1}dg_0)U - iU^{-1}dU. \] (84)
where \(-ig_0^{-1}dg_0 = dx^aP_a\). We have

\[ U^{-1}P_aU = (\cosh V)_a^bP_b + \phi^c_a\left(\frac{\sinh \tilde{V}}{\tilde{V}}\right)_c^{b'} P_{b'} = \Phi_a^bP_b, \] (85)

\[ U^{-1}P_a'U = -\phi^c_{a'}\left(\frac{\sinh V}{V}\right)^b_c P_b + (\cosh \tilde{V})_{a'}^{b'} P_{b'} = \Phi_{a'}^bP_b \] (86)
where
\[ (V^2)_a^b = -\phi^c_a\phi^b_{c'} = -(\phi\phi^T)_a^b, \quad (\tilde{V}^2)_{a'}^{b'} = -\phi^c_{a'}\phi^b_{c'} = -(\phi^T\phi)_{a'}^{b'} \] (87)
where \((\phi^T)_{c'}^{b} = \phi_{b'}^{c} = \eta^{ba}\phi_{d'}^{a}\eta_{d'c'}\). We also use
\[ (V^2)_a^b\phi_{c'}^{c'} = \phi_{a'}^{b'}(\tilde{V}^2)_{b'}^{c'}. \] (88)

The Lorentz transformation is given by
\[ \Phi = \left( \begin{array}{cc} \Phi_a^b & \Phi_{a'}^{b'} \\ \Phi_{a'}^{b} & \Phi_{a'}^{b'} \end{array} \right) = \left( \begin{array}{cc} (\cosh V)_a^b & \phi^c_a\left(\frac{\sinh \tilde{V}}{\tilde{V}}\right)_c^{b'} \\ -\phi^c_{a'}\left(\frac{\sinh V}{V}\right)^b_c & (\cosh \tilde{V})_{a'}^{b'} \end{array} \right) \] (89)
. We can write the Lorentz transformations as

\[ \Phi = \begin{pmatrix} I_1 & \varphi \\ -\varphi^T & I_2 \end{pmatrix} \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} = \begin{pmatrix} B_1 & \varphi B_2 \\ -\varphi^T B_1 & B_2 \end{pmatrix} \] (90)

where \( B_1 = (I_1 + \varphi \varphi^T)^{-\frac{1}{2}} \) and \( B_2 = (I_2 + \varphi^T \varphi)^{-\frac{1}{2}} \). In this equation \( \varphi \) carries the indices \( \varphi_{a^b} \) and so the matrices \( B_1 \) and \( B_2 \) have the indices \( (B_1)_{a^b} \) and \( (B_2)_{a'^b'} \) respectively and similarly for the unit matrices \( I_1 \) and \( I_2 \). The inverse Lorentz transformation is given by

\[ \Phi^{-1} = \begin{pmatrix} B_1 & -B_1 \varphi \\ B_2 \varphi^T & B_2 \end{pmatrix} \] (91)

\[ (B_1)_{a^b} = (\cosh V)_{a^b}, \quad (B_2)_{a'^b'} = (\cosh \tilde{V})_{a'^b'} \] (92)

\[ (\varphi)_{a'^b'} = \phi_{a^c'} (\frac{\sinh \tilde{V}}{\tilde{V} \cosh \tilde{V}})^{b'}_{c'}, \quad (\varphi^T)_{a'^b} = (\varphi)_{b^a'}. \] (93)

Using these equations the Cartan forms associated to the translations are

\[ e^a = dx^b \Phi_b^a = dx^b (\cosh V)_{b^a} - dx^{b'} \phi_{b'}^c (\frac{\sinh V}{V})_{c^a}, \] (94)
\[ e^{a'} = dx^b \Phi^a_b = dx^b \phi^c_{b'} \left( \frac{\sinh \tilde{V}}{\tilde{V}} \right)^c_{a'} + dx^{b'} (\cosh \tilde{V})_{b'}^{a'}, \quad (95) \]

We therefore take as our action
\[ S^{NG} = \int (e^0 \wedge e^0 \wedge \ldots e^0 \wedge)^* = \int d^{p+1} \xi \det e_{i}^{a}. \quad (96) \]

The action of equation (2.12) is a functional of \( \varphi_{a}^{b'} \) and \( x^{b} \). Using the expression of equation (2.10) we find that we may write the action as
\[ \int d^{p+1} \xi \det \left( \frac{\partial x^c}{\partial \xi^i} \right) \det(I_1 - \partial x \varphi^T) \det(I_1 + \varphi \varphi^T)^{-\frac{1}{2}} \quad (97) \]

where \( \partial x \) is the matrix \( (\partial x)_{a}^{b'} = \frac{\partial x^{b'}}{\partial x^a} \). Varying with respect to \( \varphi_{a}^{b'} \) we find the equation of motion
\[ (I_1 + \varphi \varphi^T)^{-1} \varphi = -(I_1 - \partial x \varphi^T)^{-1} \partial x . \quad (98) \]

In deriving this equation we have used the fact that \( (I_1 + \varphi \varphi^T) \) is a symmetric matrix. Multiplying by \( \varphi^T \) we find that
\[ (I_1 + \varphi \varphi^T)^{-1} = (I_1 - \partial x \varphi^T)^{-1} \quad (99) \]

Substituting this result in equation (2.14) we conclude that
the unique solution is

$$\varphi = -\partial x, \quad \text{or} \quad \varphi_{a}^{\; b} = -\frac{\partial x^{b}}{\partial x^{a}}. \quad (100)$$

This latter equation is just solution to the condition

$$e_{i}^{a'} = 0. \quad (101)$$

We could have adopted this condition as an inverse Higgs condition from the outset as it is an invariant condition under the transformations of the non-linear realisation. We find that

$$e_{i}^{a} = (\partial_{i}x^{b})((I_{1} + \partial x(\partial x)^{T})^{\frac{1}{2}})_{b}^{a} \quad (102)$$

and the action (2.12) becomes the covariant Nambu-Goto action in flat space

$$A = -T \int d^{p+1}\xi \sqrt{-\det G_{ij}} \quad (103)$$

where $G_{ij}$ is the induced metric

$$G_{ij} = \partial_{i}x^{a}\partial_{j}x^{b}\eta_{ab}. \quad (104)$$
Conclusions and Open Problems

- Non-linear realizations of space-time groups allow to construct in a unified brane actions in several background spaces
- Gauge symmetries can understood as local right actions associated to unbroken translations
- Tensionless branes
- D-brane actions
- Branes in general backgrounds
- N branes and non-linear realizations