Licia Verde

A taste of cosmology

Lecture 5

INFLATION and outlook for the future
http://icc.ub.edu/~liciaaverde/CFcosmo.html
Remember

Last Judgment, Vasari, Florence Duomo
The standard cosmological model

96% of the Universe is missing!!!

Major questions:
Questions that can be addressed exclusively by looking up at the sky

1) What created the primordial perturbations?

2) What makes the Universe accelerate?

These questions may not be unrelated
Successes of the Big Bang model

GR+cosmological principle

Hubble’s law
CMB
Abundance of light elements
.. And problems

Flatness problem
Horizon problem
Monopole problem

Origin of perturbations
Flatness problem

Horizon problem

Structure Problem
The flatness problem

\[ 1 - \Omega = - \frac{c^2k}{R_0^2a^2H^2} \quad \text{Remember Friedmann equations?} \]

today \quad |1 - \Omega_0| < 0.01

\[ 1 - \Omega_0 = - \frac{kc^2}{R_0^2H_0^2} \quad 1 - \Omega = - \frac{kc^2}{R_0^2a(t)^2H(t)^2} = 1 - \Omega(t) = \frac{H_0^2(1 - \Omega_0)}{H^2(t)a^2(t)} \]

For most of the life of the universe: matter+radiation only

\[ \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} \quad \rightarrow \quad 1 - \Omega(t) = \frac{(1 - \Omega_0)a^2}{\Omega_{r,0} + \Omega_{m,0}a} \]

Matter domination \quad a \sim t^{2/3} \quad 1 - \Omega(t) = \frac{1 - \Omega_0}{\Omega_{m,0}} \left( \frac{t}{t_0} \right)^{2/3}

Universe was flatter in the past!

Exercise: \quad @t_{eq} \quad z_{m,r,eq} \sim 3000? \\
Radiation dominated \quad 1 - \Omega(t) \sim a^2 \sim t \quad 1 - \Omega(t) = (1 - \Omega(t_{eq})) \frac{t}{t_{eq}}

Exercise: \quad @t = 1s?
Very quickly you find that early on $|1-\Omega|<10^{-60}$

It is like balancing a pencil on its tip, coming back years later and still finding it balancing on its tip.
The Horizon problem

Different type of Horizons in Cosmology

Since light travels at a final speed & Universe is expanding

PARTICLE HORIZON: Maximum comoving distance light can have propagated in ti to tf

\[ d_p(t) = \int_{t_i}^{t_f} \frac{dt}{a(t)} \]

Accounts for all the past expansion history

Given the same dt, dp can be very different if a(t) is weird!

HUBBLE HORIZON: \( c/H \) or \( c/t_0 \).

For normal expansion histories \( dp \sim c/H \)

For weird ones BEWARE!
The Horizon problem

The universe is homogeneous and isotropic on large scales

Consider 2 antipodal points they are separated by $2d_H \sim 30000$ Mpc

$$d_p(t_0) = c \int_{t_{cmb}}^{t_0} \frac{dt}{a(t)} \sim d_H(t_0); \quad t_{cmb} << t_c$$

What was the Hubble radius (horizon) back then?

$$H(z)^2 \sim H_0^2 \Omega_{m,0} (1 + z)^3 \rightarrow \frac{c}{H_{z=1000}} \simeq 0.2\,\text{Mpc}$$

Only points at $\sim 0.2\,\text{Mpc}$ could have “talked”
Small aside

\[ \theta_H(z_{CMB}) = \frac{d_H}{d_A} = \frac{d_H}{d_p/(1 + z)} \sim 1^\circ \]

\[ c_s \sim c \]

Compare with the size of the CMB spots!
The last scattering surface can be divided in 40000 patches of degree size i.e. 40000 “horizons”….

How could they all be at 2.726K?

Imagine 40000 students taking an exam, and all returning THE SAME exam down to the commas…
Our Hubble radius at decoupling

\[ T_{\text{dec}} = 0.3 \text{ eV} \]

Universe expansion \((z = 1100)\)

Our observable universe today

\[ T_0 = 3 \text{ K} \]

\[ T_1 = T_2 \]
The monopole problem

Phase transitions (water freezing, Magnetization...)

Temperature of universe

- $10^{32}$ K
- $10^{27}$ K
- $10^{15}$ K
- $10^{13}$ K
- 3 K

Time after Big Bang

- $10^{-43}$ s
- $10^{-35}$ s
- $10^{-12}$ s
- $10^{-6}$ s
- $5 \times 10^{17}$ s (now)
Defects….

Domain walls

Cosmic strings

monopoles

Example from a nematic liquid crystal

The most dangerous ones are the monopoles…
The fact that we are here means that at max there is one monopole in the entire observable universe….
And where the heck these perturbations come from?
INFLATION to the rescue

Started in the 1980 with A. Guth, active research area. Still a “paradigm” not a proven fact…

Postulate a period of accelerated expansion at the very beginning! Something like a cosmological constant

Different scenarios, particle-physics motivated (?)
Why is the Universe so BIG?

Inflation to the rescue: an **accelerated expansion**

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\epsilon + 3P) \quad \text{take } P = -\epsilon \text{ for simplicity}
\]

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \rightarrow a \propto e^{H_\Lambda t}
\]

\[t_i\quad \text{Inflation begins}\]

\[t_f\quad \text{Inflation ends}\]

\[
a(t) = \begin{cases} 
  a_i \left(\frac{t}{t_i}\right)^{1/2} & t < t_i \\
  a_i e^{H_i(t-t_i)} & t_i < t < t_f \\
  a_i e^{H_i(t_f-t_i)} \left(\frac{t}{t_f}\right)^{1/2} & t > t_f 
\end{cases}
\]

\[
\frac{a(t_f)}{a(t_i)} = e^N \quad N = H_i(t_f - t_i) \quad \text{number of e- foldings}
\]

**Example:**

\[t_i = T_{GUT} = 10^{-36}s \quad H_i \sim t_{GUT}^{-1} \sim 10^{36}s^{-1} \quad N \sim 100
\]

Even if inflation lasts for

\[\Delta t \sim 10^{-34}s
\]

\[
\frac{a(t_f)}{a(t_i)} \sim 10^{43} \quad \text{NOT BAD!}
\]
Any weirdness like monopole
Would be diluted away!
Inflation solves the Flatness problem

Something like a $\Lambda$

$$H = \text{const}; \quad a \sim \exp(H_I t)$$

$$1 - \Omega(t) = \frac{c^2}{R_0^2 a^2 H^2}$$

$$\frac{a(t_f)}{a(t_i)} = e^N \quad N = H_i (t_f - t_i) \quad \text{number of e-foldings}$$

$$|1 - \Omega(t_f)| = \exp(-2N) |1 - \Omega(t_i)|$$

For N~100 NOT BAD!
What made the Universe so flat?
What made the Universe so big?
What made the universe so uniform?
What seeded the galaxies?
Why is the Universe so uniform?

Inflation solves that
Inflation solves the Horizon problem

In an accelerated expansion the particle horizon and the Hubble horizon can be VERY DIFFERENT

Given the same $dt$, $dp$ can be very different if $a(t)$ is weird!
As an added bonus:

Inflation generates Gaussian perturbations: quantum fluctuations stretched to become classical by the expansion. The mechanism is similar to Hawkings radiation. (The event horizon being the complementary concept to the particle horizon…)

Turns out that the power spectrum of these perturbations will be a power law: different model of inflation predict slightly different power laws, but all close to “scale invariant”..
Perturbations outside the Horizon?

Inflation solves that

Fluctuations frozen in
CAN THIS BE TESTED?

flatness

Power law primordial power spectrum
  Can even hope to distinguish specific models

Super horizon fluctuations (polarization)
  Coming next

gaussianity
  Can even hope to distinguish specific models

Stochastic background of gravity waves (polarization)
Cosmic History / Cosmic Mystery

Planck Energy

- Generation of primordial perturbations

GUT symmetry

- Cosmic Microwave Background Emitted carries signature of acoustic oscillations and potentially primordial gravitational waves

T=100 TeV (ILC X 100)

- non-linear growth of perturbations: signature on CMB through weak gravitational lensing

nucleosynthesis

- McMahon adapted by Peiris
There’s more!

Polarization

First detected by DASI in 2002

WMAP (2006)
Generation of CMB polarization

- Temperature quadrupole at the surface of last scatter generates polarization.

FROM Wayne Hu

YES, there is also reionization

Rees 68, Coulson et al ‘94
..... Hu& White 97 (pedagogical)
Polarization for density perturbation

- Radial (tangential) pattern around hot (cold) spots.
And it has been seen!
Komatsu, WMAP7yrs team (2010)
And it has been seen!

Planck overview paper (2013)

Observed

Theory prediction
Large-scale TE anti correlation

Density mode

Velocities (hot to cold)

Hot due to doppler

During decoupling

And it has been seen (2003)
Large-scale TE anti correlation

During decoupling

Density mode

Hot due to doppler

Velocities (hot to cold)

Gravity waves (tensor) are different...
Gravity waves stretch space...
... and create variations

Image from J. Rhul.
E and B modes polarization

E polarization
from scalar, vector and tensor modes

B polarization only from (vector) tensor modes

Kamionkowski, Kosowsky, Stebbings 1997, Zaldarriga & Seljak 1997
Information about the shape of the inflaton potential is enclosed in the shape and amplitude of the primordial power spectrum of the perturbations.

Information about the energy scale of inflation (the height of the potential) can be obtained by the addition of B modes polarization amplitude.

In general the observational constraints of $N_{	ext{fold}} > 50$ requires the potential to be flat (not every scalar field can be the inflaton). But detailed measurements of the shape of the power spectrum can rule in or out different potentials.
The inflationary solution

$V(\phi)$

$H \sim \text{const}$

Accelerated expansion...

Factor $10^{26}$ in less than $10^{-34}$ s

Solves cosmological problems (Horizon, flatness).

Cosmological perturbations arise from quantum fluctuations, evolve classically.

Perturbations: adiabatic

( isocurvature )

Photon Neutrinos
Baryons (+electrons)
Dark matter
(dark energy negligible early on)

Tensor (gravitational waves)

Fig. From W. Hu.
Inflationary cosmology

Friedmann equations, $a$ is scale factor

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3M_{pl}^2} \rho, \quad = \frac{1}{3M_{pl}^2} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) \]

\[ \dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2} (\rho + 3p) = -\frac{1}{3M_{pl}^2} (\ddot{\phi}^2 - V(\phi)) \]

\[ \ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0. \]

Single field slow roll inflation: the simplest example

Inflate: \[ V \gg \dot{\phi}^2. \]

\[ a(t) \approx a(0)e^{Ht}, \quad H \approx \text{const}. \]

Sustain it \[ |\ddot{\phi}| \ll |V'|. \]

Process must terminate somehow

Restrictions in the form of the inflaton potential $V$:

SLOW ROLL PARAMETERS

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2}{2} \frac{\dot{\phi}^2}{H^2} \approx \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2, \]

\[ |\eta| \approx M_{pl}^2 \left| \frac{V''}{V} \right|. \]
Inflationary cosmology

Physical wavelength of fluctuations is stretched by expansion
The physical horizon is time dependent

Physical wavelengths grow faster than the horizon
Quantum fluctuations get stretched to become classical and “super-horizon” because of the accelerated expansion

But the spacing of the fluctuations (their power as a function of scale) depend on how fast they exited the horizon (H)

Which in turns depend on the inflaton potential

The shape of the primordial power spectrum encloses information on the shape of the inflaton potential!
Fingerprints of the early universe

$V(\phi)$

Small scales  Solar System  Horizon  large scales

$V, V', V'', V'''$

CMB  LSS  21 cm?  GWO?
Primordial power spectrum $= A k^n$

Amplitude of the power law

Equal power per log $k$, $n=1$, scale invariant
Slow roll predictions

\[ P_s(k) = \frac{1}{24\pi^2 M_{pl}^4} \frac{V}{\epsilon} \Bigg|_{k=aH}, \quad n_s - 1 = 2\eta - 6\epsilon, \]

\[ P_t(k) = \frac{2}{3\pi^2 M_{pl}^4} \frac{V}{\epsilon} \Bigg|_{k=aH}, \quad n_t = -2\epsilon, \quad r = 16\epsilon. \]

\[ V(\phi) = V|_* + V'|_*(\phi - \phi_*) + \frac{1}{2} V''|_*(\phi - \phi_*)^2 + \frac{1}{3!} V'''|_*(\phi - \phi_*)^3 + \cdots. \]

\[ \varepsilon \propto V'/V \]

Other models, of course
CMB Consistent with Simplest Inflationary Models

- Flat universe: \( \Omega_{\text{tot}} = 0.9895 \pm 0.045 \) (95\% CL)

- Gaussianity: \( f_{NL} = 2.7 \pm 5.8 \)

- Power Spectrum spectral index nearly scale-invariant (red): \( n_s = 0.9603 \pm 0.0073 \)

- Adiabatic initial conditions

- Superhorizon fluctuations (TE anticorrelations)

Still testing basic aspects of inflationary mechanism rather than specific implementations

Causal Seed model (Durrer et al. 2002)
Primordial Isocurvature i.c.

TE data

Primordial Adiabatic i.c.

Hu & Sujiyama 1995
Zaldarriaga & Harari 1995
Spergel & Zaldarriaga 1997
Peiris et al 2003
Specific models critically tested

Models like $V(\phi) \sim \phi^p$

For 50 and 60 e-foldings

$p=4,3$
We happen to live in a galaxy!
K Band (23 GHz)

Dominated by synchrotron; Note that polarization direction is perpendicular to the magnetic field lines.

Page et al 2007
Ka Band (33 GHz)
Synchrotron decreases as $n^{-3.2}$ from K to Ka band.
Q Band (41 GHz)

We still see significant polarized synchrotron in Q.

Page et al 2007
V Band (61 GHz)
The polarized foreground emission is also smallest in V band. We can also see that noise is larger on the ecliptic plane.

Page et al 2007
W Band (94 GHz)

While synchrotron is the smallest in W, polarized dust (hard to see by eye) may contaminate in W band more than in V band.

Page et al 2007
The next frontier: gravity waves

Verde Peiris Jimenez 05
Windows into the primordial Universe

Recombination  380,000 yrs  Atomic physics/GR

Nucleosynthesis  3 minutes  Nuclear physics

LHC

inflation  $10^{-30}$ s (?)  TeV energies

Big BANG
What next?

Polarization, the next frontier

Why measure CMB Polarization?

Directly measures dynamics in early universe

So far:
Critical test of the underlying theoretical framework for cosmology

Future: “How did the Universe begin?”
Improve cosmological constraints
Eventually, perhaps, test the theory of inflation.

Plans for the ultimate primary polarization CMB experiment
Inflation consists of taking a few numbers that we don’t understand and replacing it with a function that we don’t understand.

David Schramm 1945 - 1997
Probing cosmic structures and radiation with the ultimate polarimetric spectro-imaging of the microwave and far-infrared sky.

Find out more and offer your support at http://www.prism-mission.org

No, not THAT PRISM.
PRISM
Key concepts

Inflation:
  the flatness problem, horizon problem, structure problem and monopole problem

CMB polarization