Licia Verde

a taste of Cosmology

Lecture 2

Friedmann Equations
The content of the Universe
(Cosmological parameters; intro)

http://icc.ub.edu/~liciaverde/cernlectures.html

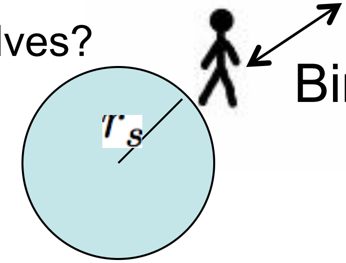
- In GR space tells mass how to move, mass tells space how to curve.

- Suspicion: a(t) related to content of Universe?
- Really need GR but we do Newton…

The theory that really describe the Universe at the largest scales must be GR.
Dwell a bit on Einstein vs Newton…

Friedmann equations (1)

Take a uniform sphere, how does a point on its surface evolves?



Birkhoff theorem

$$a_{cc} = \frac{d^2 r_s(t)}{dt^2} = -\frac{GM_s}{r_s^2(t)}$$

$$\frac{dr_s}{dt}\frac{d^2 r_s(t)}{dt^2} = -\frac{GM_s}{r_s^2(t)}\frac{dr_s}{dt} = \frac{1}{2}\frac{d}{dt}\left(\frac{dr_s}{dt}\right)^2$$

integrate

$$\frac{1}{2}\left(\frac{dr_s}{dt}\right)^2 = \frac{GM_s}{r_s(t)} + U$$

kinetic potential

$$Ms = \frac{4\pi}{3}\rho(t) r_s^3(t) \equiv \frac{4\pi}{3}\rho(t) a^3(t) r_s^3$$

Substitute and divide each side by $\frac{r_s^2 a^2}{2}$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) + \frac{2U}{r_s^2 a^2}$$

Symmetric under time reversal

Start with an expanding sphere:
How does it evolve of U>0 ?
How does it evolve of U<0 ?

Need GR
to go further

Friedmann equations 1(continued)

$$\boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R_0^2 a^2}}$$

U was related to spatial curvature

ρ Total matter-energy density: ε/c^2

Make the Universe flat

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G}$$

Today (and at any time using (t) for 0)

Define: $$\Omega = \frac{\rho_{tot}}{\rho_c}$$

Density parameter, gives curvature

$$\text{Mass-energy} \begin{cases} \rightarrow \text{gravity} \\ \rightarrow \text{curvature} \end{cases}$$

mass tells space how to curve.

COSMOLOGICAL PARAMETERS so far we have met 2: H and now Ω

Friedmann equations 2

One eq.; 2 unknowns a(t) , ρ(t), need a relation between the two

$dQ = dE + PdV$ Expanding universe, adiabatic $\dot{E} + P\dot{V} = 0$

Consider a spherical chunk of Universe as in the sphere of the example before

$$\dot{V} = \frac{4\pi}{3}\frac{d}{dt}(ar_s)^3 = \frac{4\pi}{3}r_s^3 3a^2\dot{a} = V\left(3\frac{\dot{a}}{a}\right)$$

$$\dot{E} = \frac{d}{dt}\epsilon(t)V(t) = \dot{V}\epsilon + V\dot{\epsilon} = V\left(\dot{\epsilon} + 3\frac{\dot{a}}{a}\epsilon\right)$$

$$0 = \dot{E} + P\dot{V} = V\left(\dot{\epsilon} + 3\frac{\dot{a}}{a}\epsilon\right) + PV3\frac{\dot{a}}{a}$$

ε same as ρ

$$\boxed{\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0}$$

Fluid equation

Friedman and fluid equations are ENERGY CONSERVATION

Friedmann Equations 3

Let's combine the two to get a useful eqn

Multiply Friedmann by a^2

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\epsilon a^2 - \frac{kc^2}{R_0^2}$$

derive wrt t

$$2\dot{a}\ddot{a} = \frac{8\pi G}{3c^2}(2\epsilon a\dot{a} + \dot{\epsilon}a^2)$$

Divide by $2\dot{a}a$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3c^2}(\dot{\epsilon}\frac{a}{\dot{a}} + 2\epsilon)$$

Use fluid eq.

$$\dot{\epsilon}\frac{a}{\dot{a}} = -3\left(\epsilon + P\right)$$

$$\boxed{\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P)}$$

Acceleration equation!

If P=0 and ε>0 …

So if you start with an expanding Universe….

If the Universe started off expanding stuff should slow down the expansion!

And what would it take to make it accelerate?

Friedmann equations

NOT independent!

$$\boxed{\begin{aligned} H^2 = \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R_0^2 a^2} \\ \dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) &= 0 \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G}{3c^2}(\epsilon + 3P) \end{aligned}}$$

$$\rho(t)c^2 = \epsilon(t)$$

If you want to solve for

$$a(t), \epsilon(t), P(t)$$

need more info…

$$P = P(\epsilon)$$

$$P = w\epsilon$$

Interesting cases:

Non-relativistic matter $w \simeq 0$

R adiation $w = 1/3$

Accelerating fluid $w < -1/3$

Cosmological constant $w = -1$

This is weird……
but looks like we are stuck with it

Einstein and the cosmological constant

$\nabla^2\Phi = 4\,\pi G\rho$ Poisson equation $a_{cc} = -\vec{\nabla}\Phi \equiv 0$

If static

If static $\rho = \frac{1}{4\pi G}\nabla^2\Phi = 0$ Empty… humm….. or $\nabla^2\Phi + \Lambda = 4\pi G\rho$

$\Lambda = 4\,\pi G\rho$

Einstein….

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{R_0^2 a^2} + \frac{\Lambda}{3}$$

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

Does not get diluted… $\epsilon_\Lambda = \frac{c^2}{8\pi G}\Lambda$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3}$$

$w = -1, P_\Lambda = -\epsilon$

Vacuum energy…

I told you it was weird….

The universe composition

Matter; radiation; curvature;etc…

$$H^2 = \frac{8\pi G}{c^2 3} \sum_i \epsilon_i$$

One for each component

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\sum_i \epsilon_i + 3\sum_i P_i\right)$$

$$\begin{aligned} \dot{\epsilon} &= -3(1+w)\epsilon\frac{\dot{a}}{a} \\ d\ln\epsilon &= -3(1+w)d\ln a \\ \Aboxed{\epsilon &= \epsilon_0 a^{-3(1+w)}} \end{aligned}$$

Matter…
Radiation… (redshift is back)
Cosmological constant…

Use Friedmann equation

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\epsilon_0 a^{-(1+3w)}$$

You can solve it!

Exercise: If it was only one component

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\epsilon_0 a^{-(1+3w)}$$ Ansatz: $a \sim t^q$ If $w \neq -1$ $$q = \frac{2}{3(1+w)}$$

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$t_0 = \sqrt{\frac{c^2}{6\pi G\epsilon_0}}\frac{1}{(1+w)}$$

$$H_0 = \left.\frac{\dot{a}}{a}\right|_{t_0} = t_0^{-1}\frac{2}{3(1+w)}$$

$$(1+z) = \frac{a(t_o)}{a(t_e)} = \left(\frac{t_o}{t_e}\right)^{\frac{2}{3(1+w)}} \longrightarrow t_e(t_o)$$

$$d_p(t_o, z) = c\int_{t_e}^{t_o}\frac{dt}{a(t)} = .. = \frac{c}{H_0}\frac{2}{1+3w}\left[1-(1+z)^{-(1+3w)/2}\right]$$

The farther object you can see…. Horizon! (we' ll get back to this later)

Exercise: If it was only Λ

$$\dot{a}^2 = \frac{8\pi G}{3c^2}\epsilon_0 a^{-(1+3w)}$$

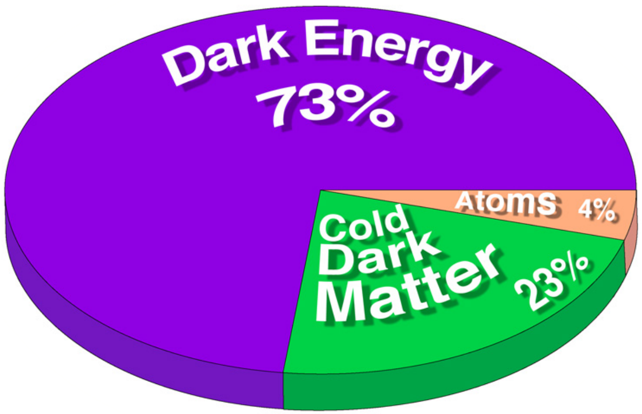
$$H = \left(\frac{8\pi G\epsilon_\Lambda}{3c^2}\right)^{1/2} = H_0$$

$$\frac{\dot{a}}{a} = const \longrightarrow a(t) = \exp[H_0(t - t_0)]$$

$$PHYSICAL: \; d_p(t) = a(t)\int \frac{dt}{a(t)} = \exp(H_\Lambda t)\int_{t_i}^{t_f} \exp(-H_\Lambda t)dt$$
$$= H_\Lambda^{-1}(\exp(H_\Lambda(t - t_i)) - 1)$$

Grows exponentially and c/H<<dp!

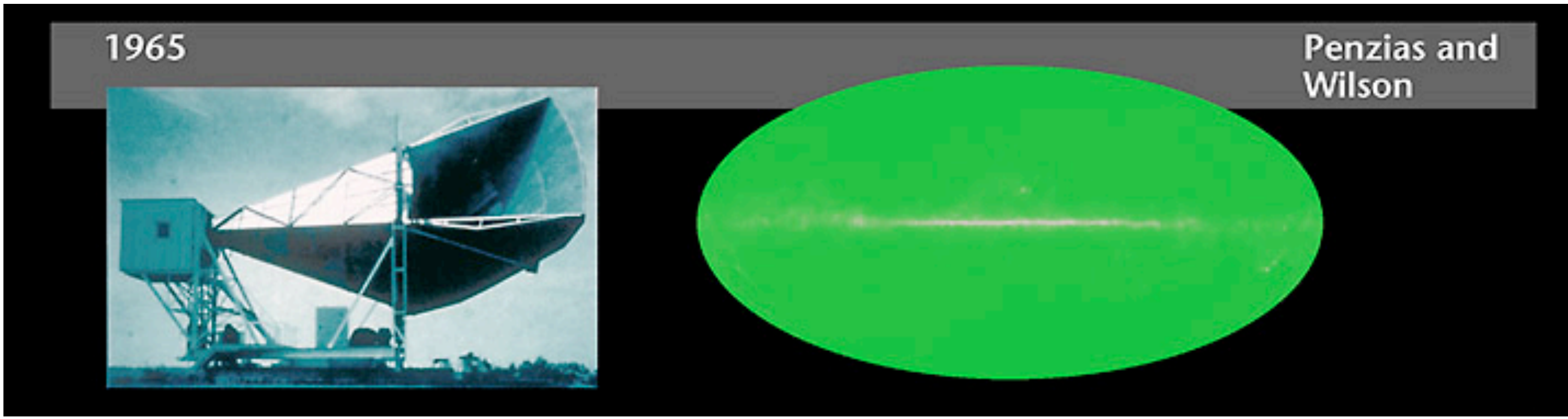
The standard cosmological model



Radiation…

TODAY WE KNOW THIS (how we got there …more in the next lectures)
All the component scale with scale factor in different ways so today the composition is this one (we' ll see how we know this later on) but in the past, was different!

Radiation



Back to Friedmann

$$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{kc^2}{R_0^2 a^2} \longrightarrow (1-\Omega) = -\frac{kc^2}{R_0^2 a^2 H_0^2}$$

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \qquad \Omega = \frac{\rho_{tot}}{\rho_c}$$

$$\frac{H(t)^2}{H_0^2} = \frac{8\pi G}{3c^2}\frac{\epsilon}{H_0^2} - \frac{(\Omega_0 - 1)}{a^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda_0} + \frac{\Omega_{k,0}}{a^2}; \qquad \Omega_k = 1 - \Omega$$

If you have a mix of components, chances are that at different times in the life of the Universe different components dominate

Back to Friedmann

$$H(t)^2 = \frac{8\pi G}{3c^2}\epsilon - \frac{kc^2}{R_0^2 a^2} \longrightarrow (1-\Omega) = -\frac{kc^2}{R_0^2 a^2 H_0^2}$$

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \qquad \Omega = \frac{\rho_{tot}}{\rho_c}$$

Cosmological parameters!

$$\boxed{\frac{H(t)^2}{H_0^2} = \frac{8\pi G}{3c^2}\frac{\epsilon}{H_0^2} - \frac{(\Omega_0 - 1)}{a^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda_0} + \frac{\Omega_{k,0}}{a^2}; \qquad \Omega_k = 1 - \Omega}$$

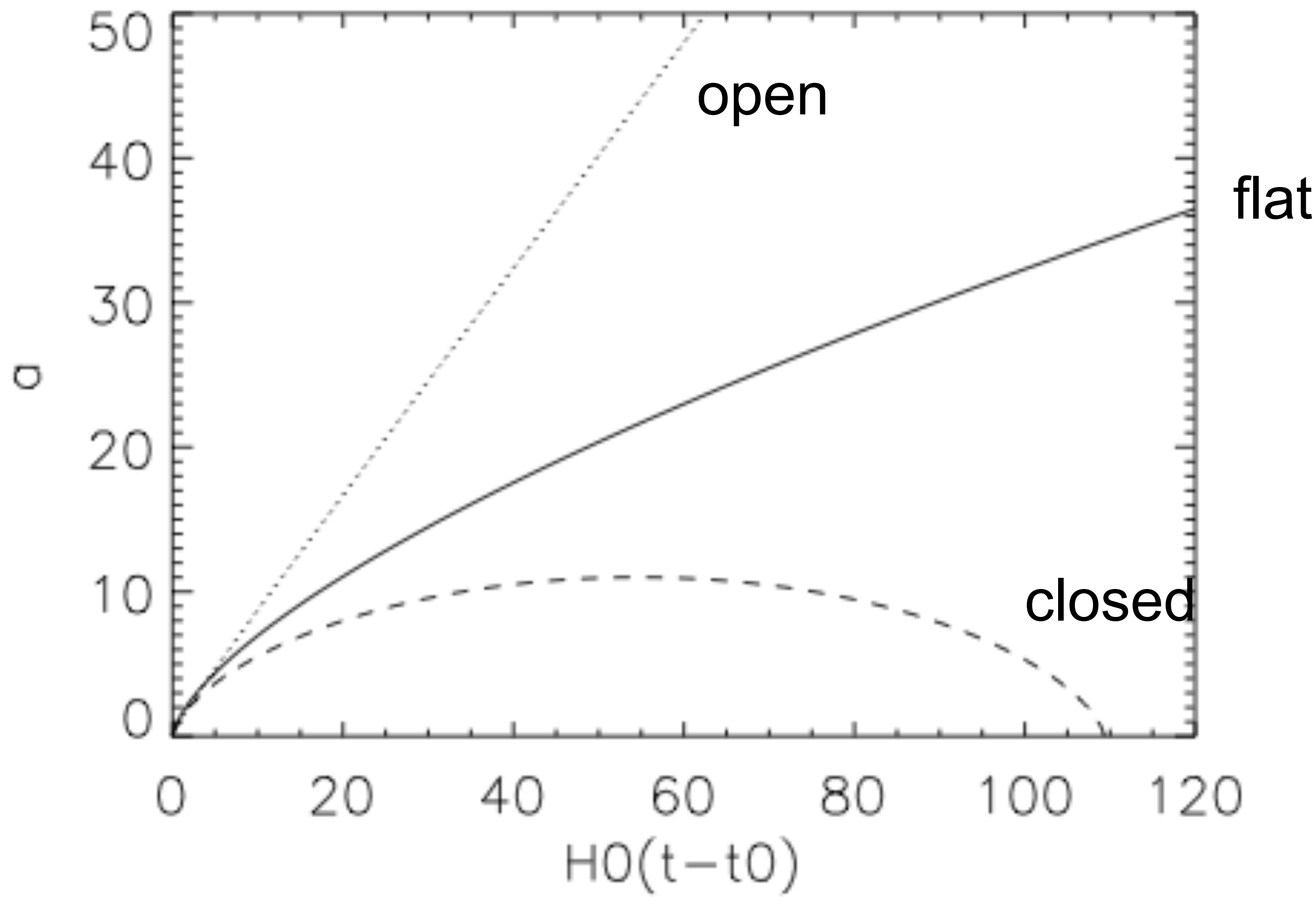
If you have a mix of components, chances are that at different times in the life of the Universe different components dominate

COSMOLOGICAL PARAMETERS!

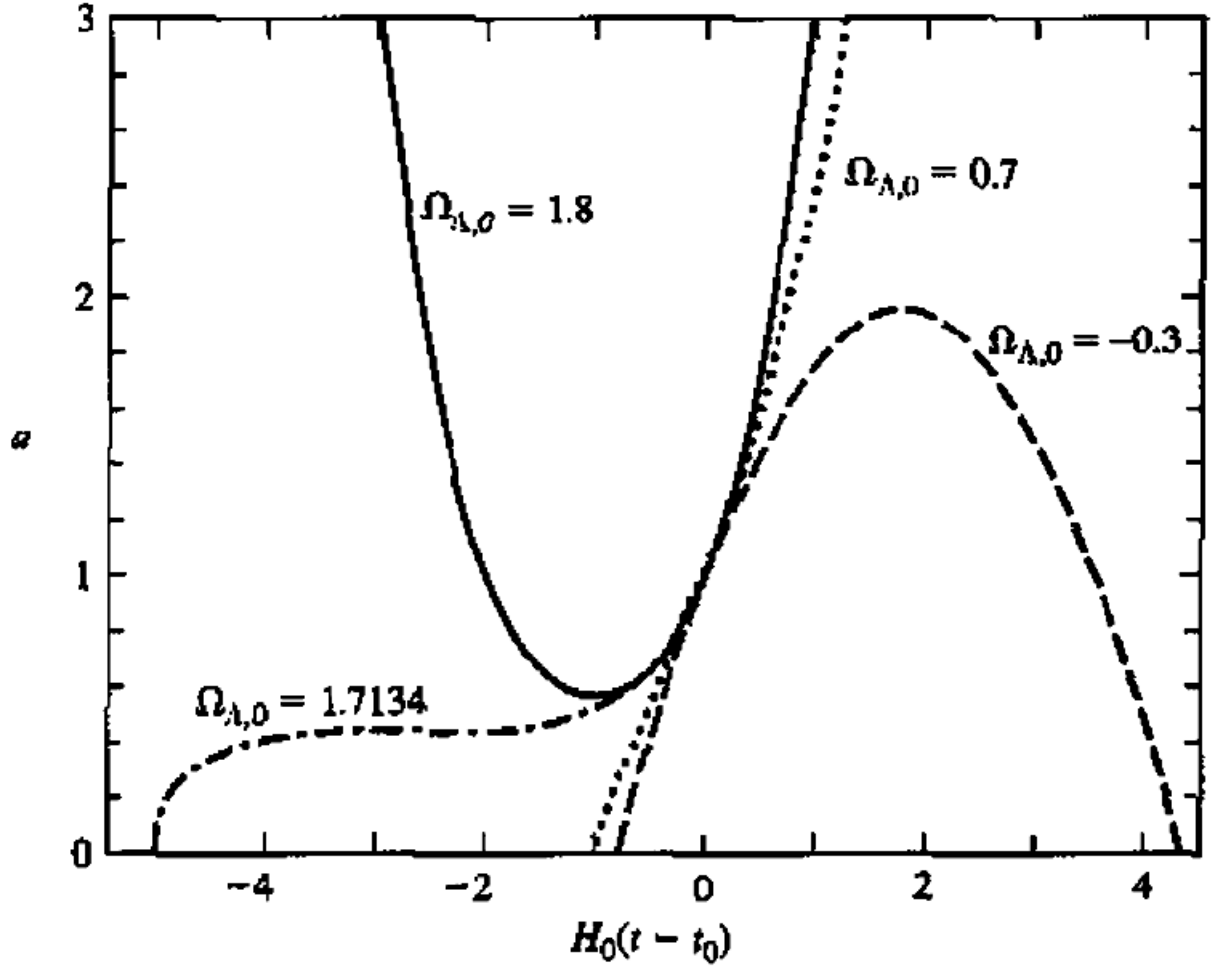
A lot of the effort in the past 15 years of cosmology was to constrain these parameters!

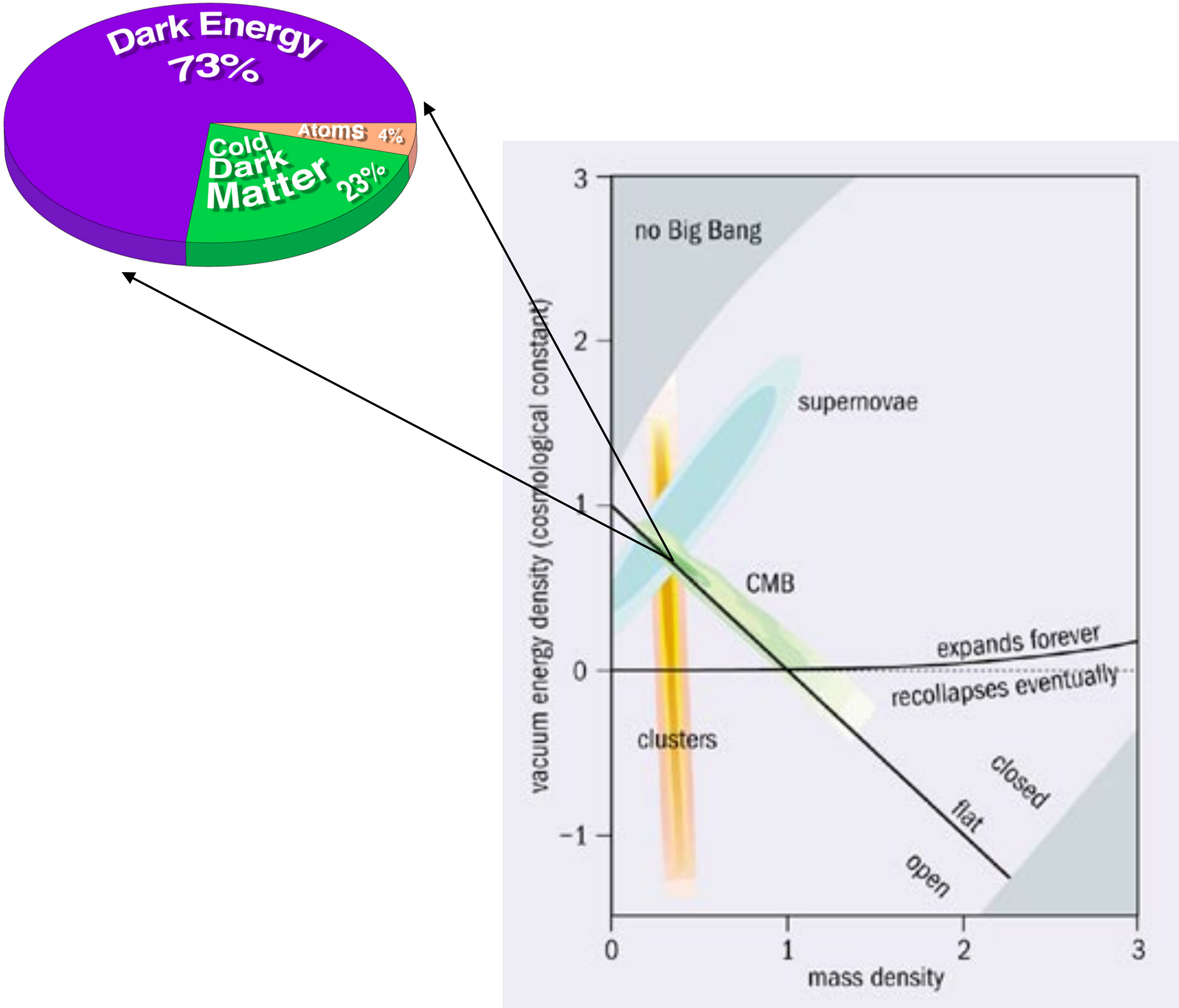
MATTER + CURVATURE ONLY

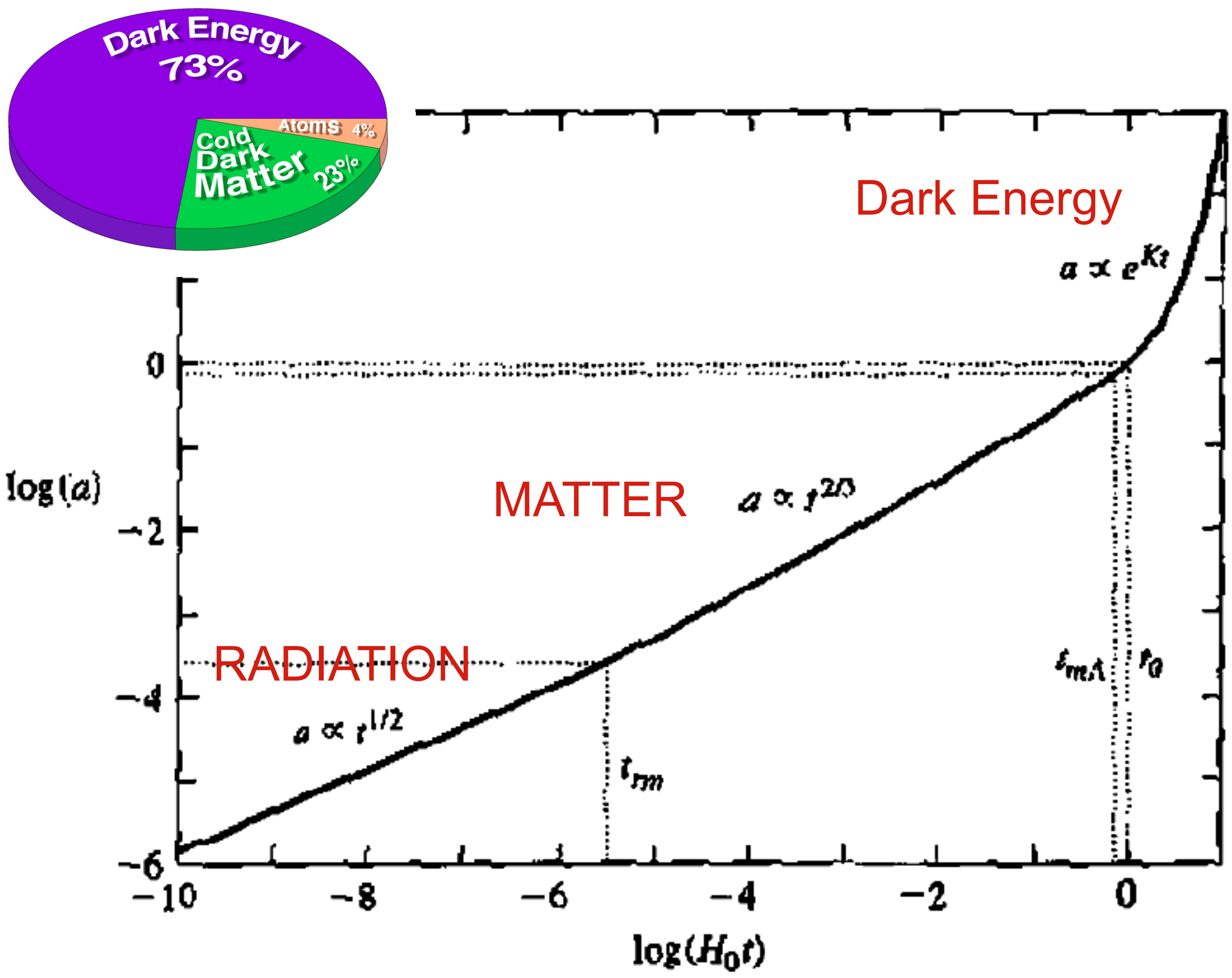
Density is destiny!!



A ZOO OF POSSIBILITIES…..

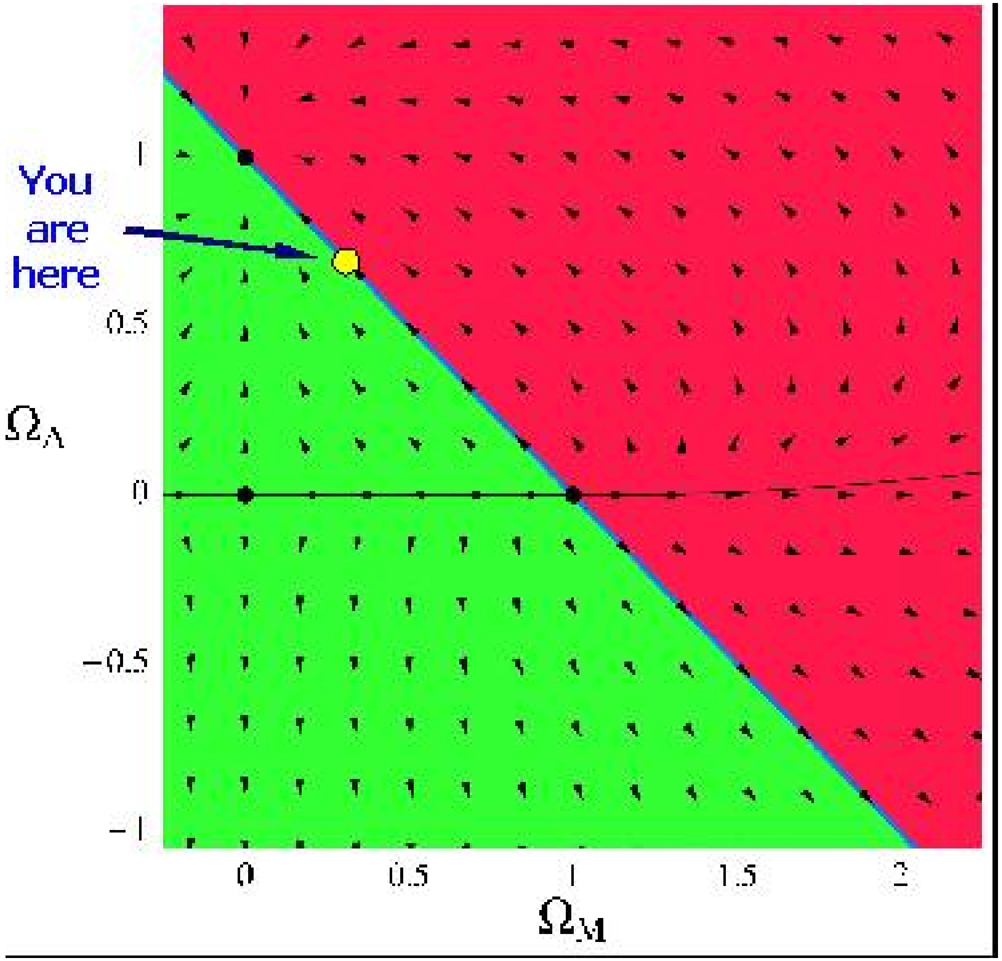



Dark Energy
73%
Atoms 4%
Cold Dark Matter
23%
no Big Bang
supernovae
CMB
expands forever
recollapses eventually
clusters
closed
flat
open
vacuum energy density (cosmological constant)
mass density
3
2
1
0
−1
0
1
2
3



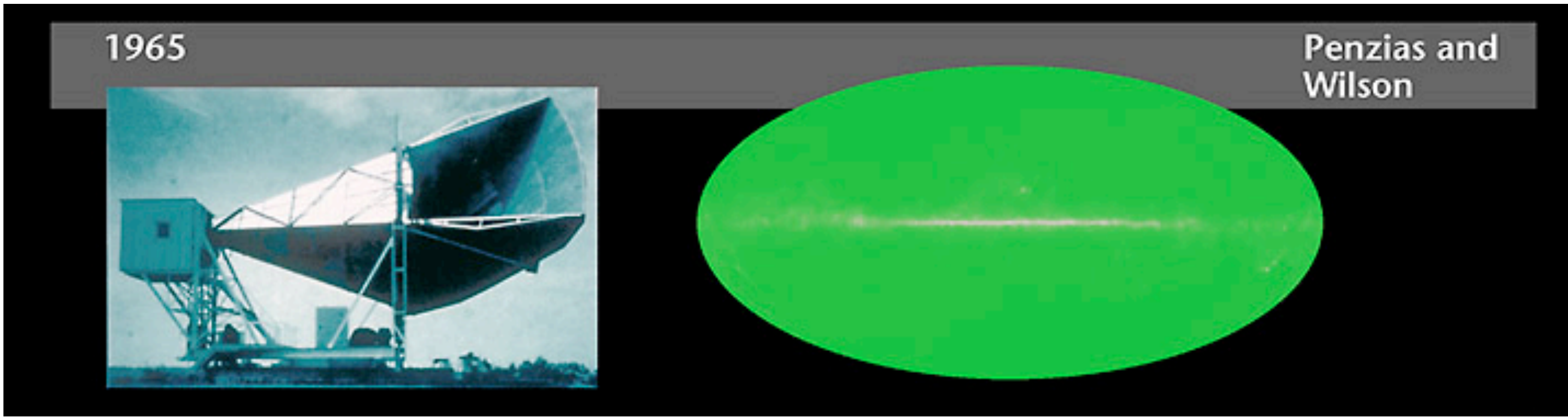
Dark Energy
73%
Atoms 4%
Cold Dark Matter 23%
Dark Energy
$a \propto e^{Kt}$
MATTER
$a \propto t^{2/3}$
RADIATION
$a \propto t^{1/2}$
t_{rm}
$t_{m\Lambda}$
t_0
log(a)
0
−2
−4
−6
−10
−8
−6
−4
−2
0
$\log(H_0 t)$

Evolution….



This is a BIG puzzle

Radiation

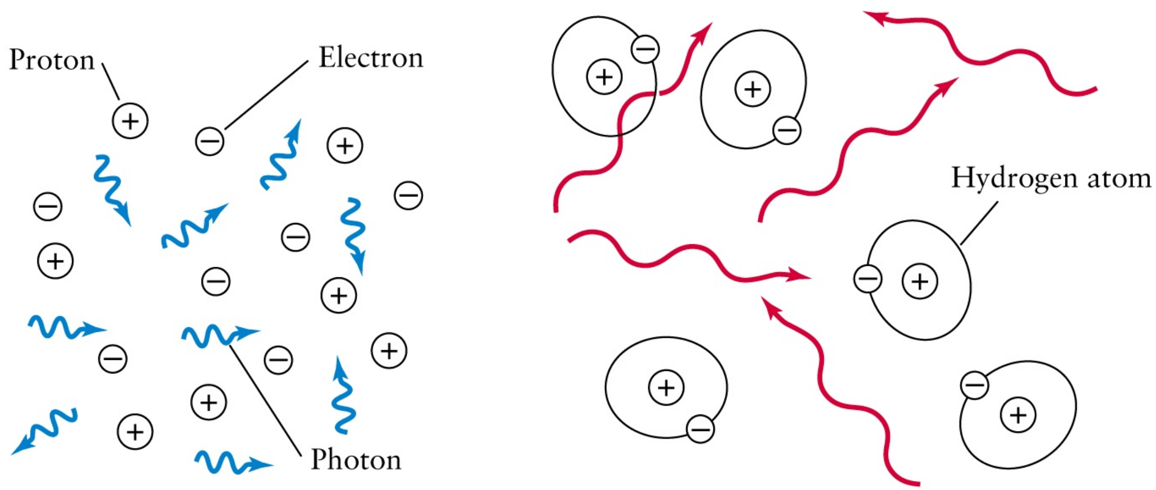


The cosmic microwave background (CMB) radiation

Regular hydrogen gas lets light pass through more or less unimpeded. This is the case today, where the hydrogen gas is either cold and atomic, or very thin, hot, and ionized.

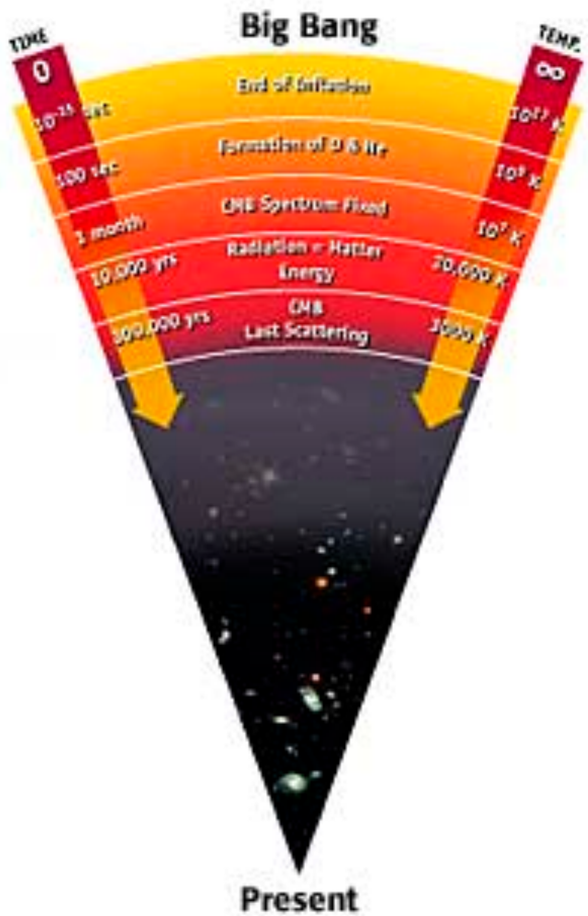
But in the early universe, when it was much warmer, the gas would have been ionized, and the universe opaque to light—as if you were in a dense fog.

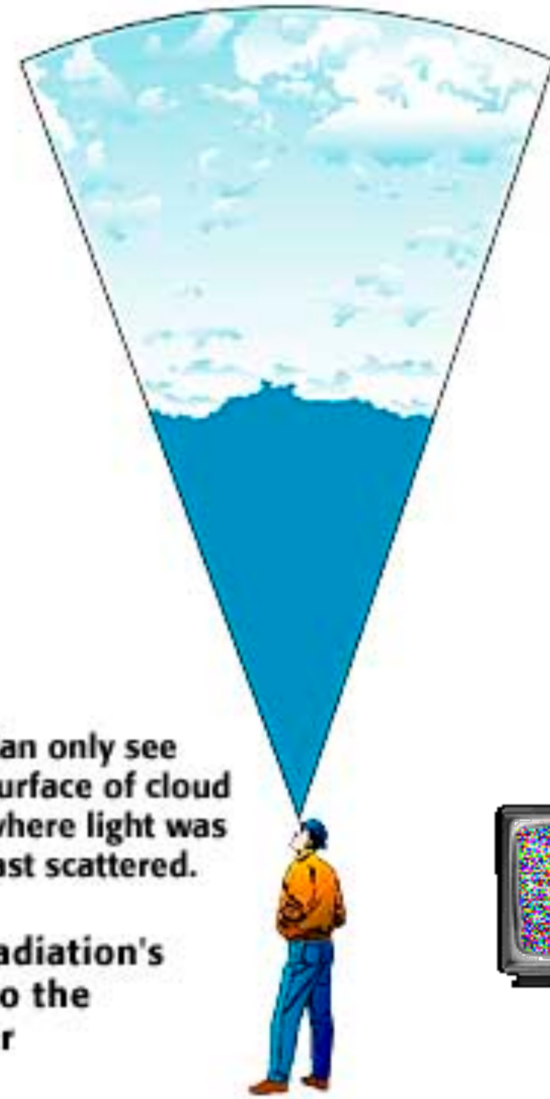
As the universe cooled, the electrons and protons "recombined" into normal hydrogen, and the universe suddenly became transparent.



a Before recombination b After recombination

The last scattering surface: a snapshot of the early universe

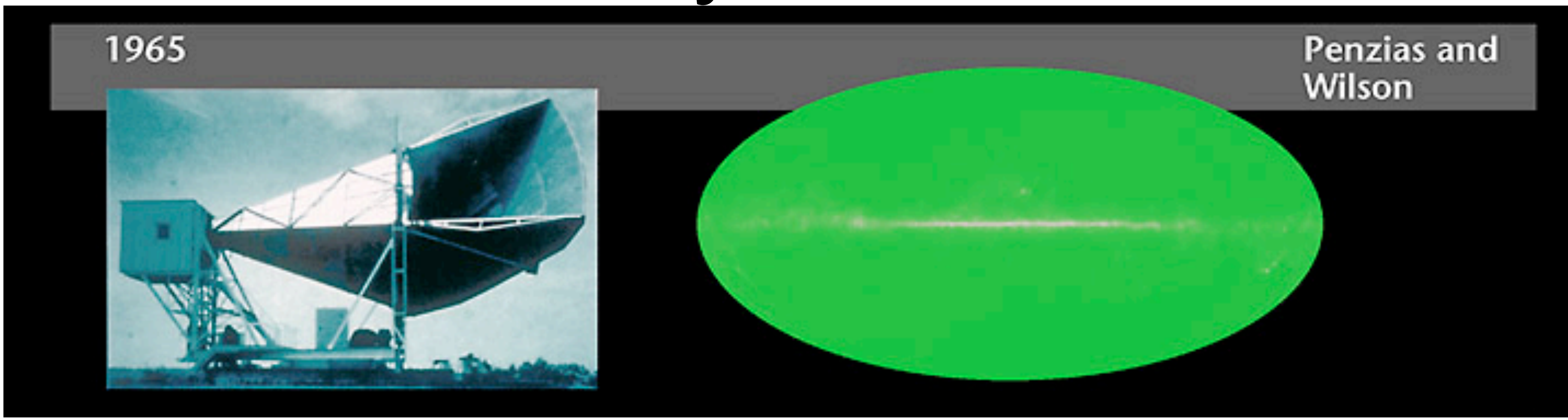




Can only see
surface of cloud
where light was
last scattered.

The Cosmic Microwave Background Radiation's
"surfact of last scatter" is analogous to the
light coming through the clouds to our
eye on a cloudy day.

Discovery of the CMB



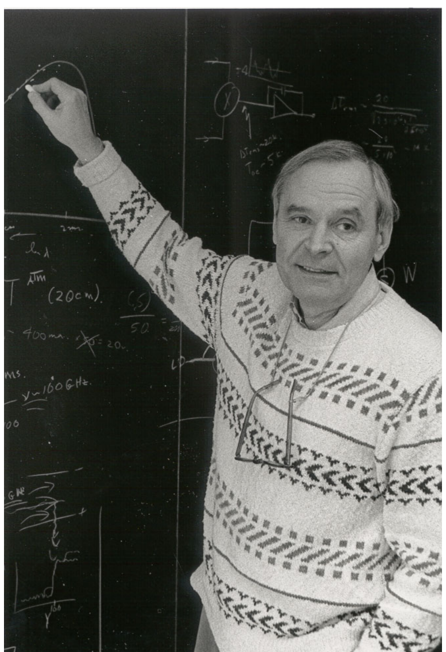
Two engineers for Bell Labs accidentally discovered the CMB radiation, as a uniform glow across the sky in the radio part of the spectrum, in 1965. It is the blackbody emission of hot, dense gas (T~3000 K, λ_{max}~1000 nm) red-shifted by a factor of 1000, to a peak wavelength of 1 mm and T~3K.

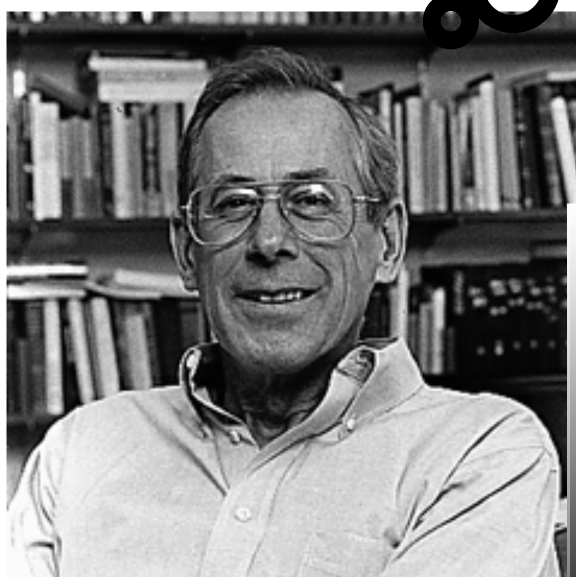
Nobel prize 1978

Some history

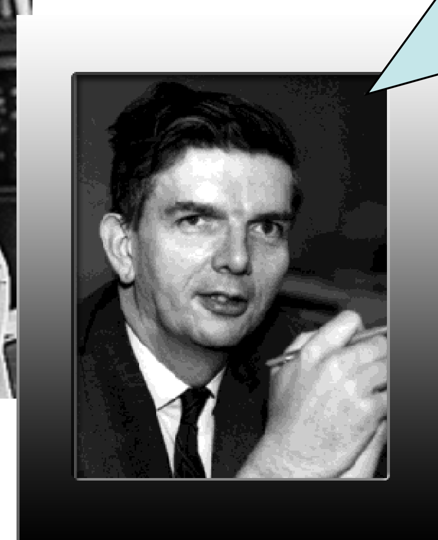
1965

Penzias & Wilson





With Roll



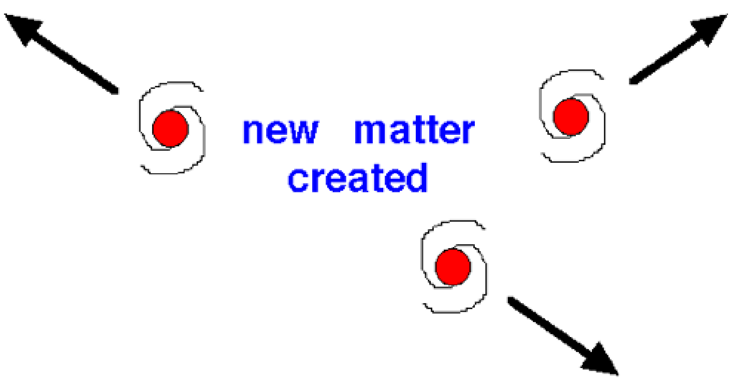
Well boys, we have been scooped

REMEMBER THIS?

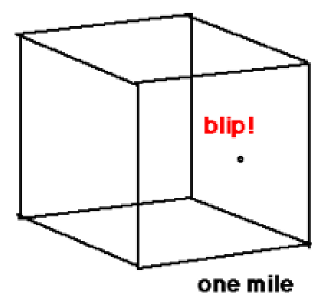
Alternatives to the big bang (historical interest only)

The steady state Universe (Fred Hoyle)

Infinitely old
Infinitely big
Constant density
Expanding (Hubble's Law)
CONTINUOUS MATTER CREATION



• expansion creates hydrogen atoms



• one hydrogen atom appears every year in a one-cubic mile volume

How to distinguish the two theories?

BigBang

SteadyState

Problems:

Cosmic singularity

Physics as we know it cease to work at small t

Unconfortable...
(what was there before?)

Does not conserve energy

We see galaxy evolution

We do not see objects older than 13 Gyr...

Formation of elements?

Prediction (example)

Density and temperature

should have been

should **not** have been

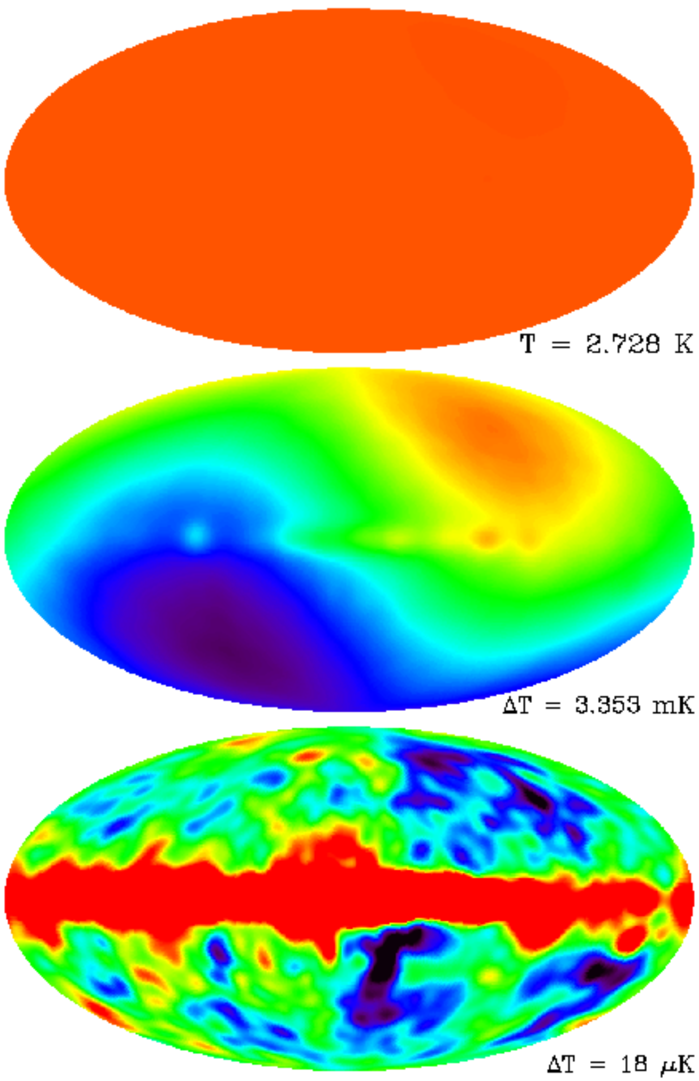
higher in the past

CMB

~~CMB~~

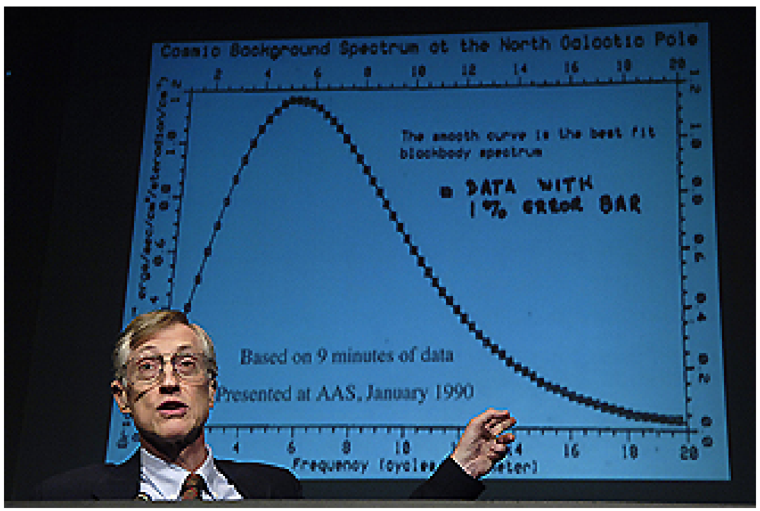
THREE IMPORTANT RESULTS

- Penzias & Wilson saw only a uniform glow…
- Later, Doppler shift from Milky Way's motion seen; the Galaxy is moving towards Hydra/ Centaurus at 620 km/ s…
- Only in 1992 was any nonuniformity in the CMB observed—and then only about 10^{-5} K worth.



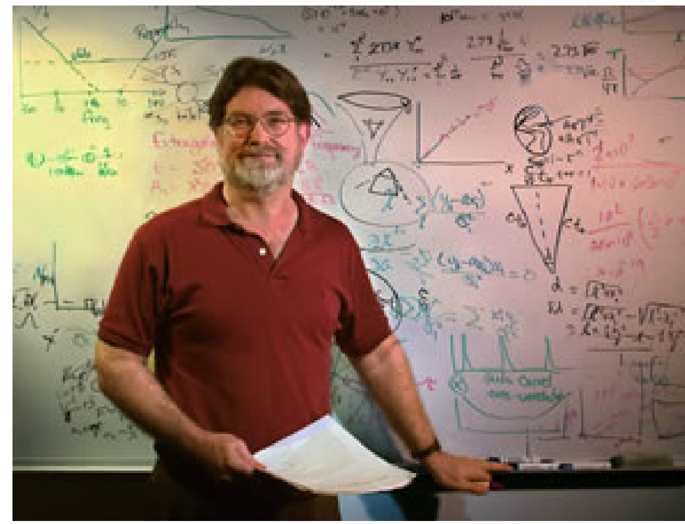
COBE: 2006 Nobel prize in Physics

“from unexpected noise to precision science”



John Mather

blackbody

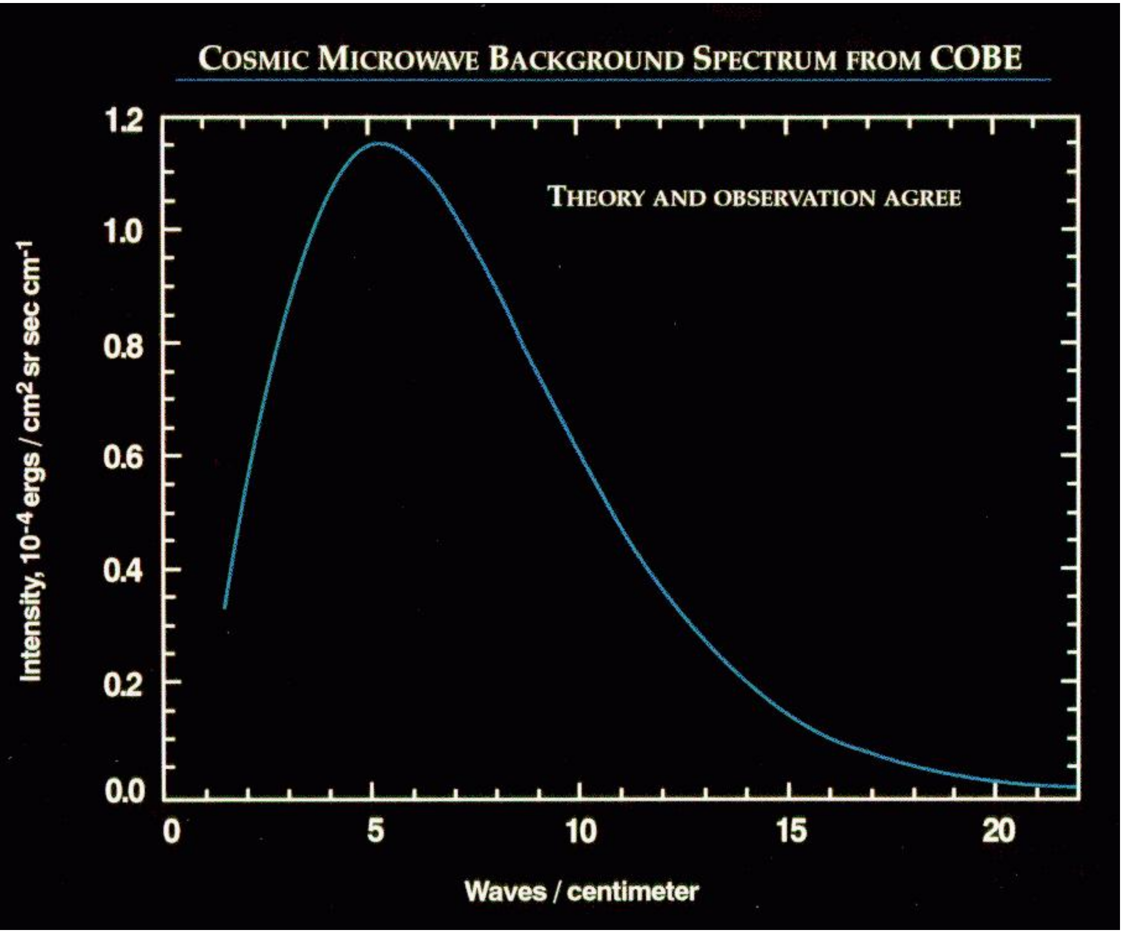


George Smoot

perturbations

The perfect blackbody!

Far Infrared Absolute Spectrophotometer

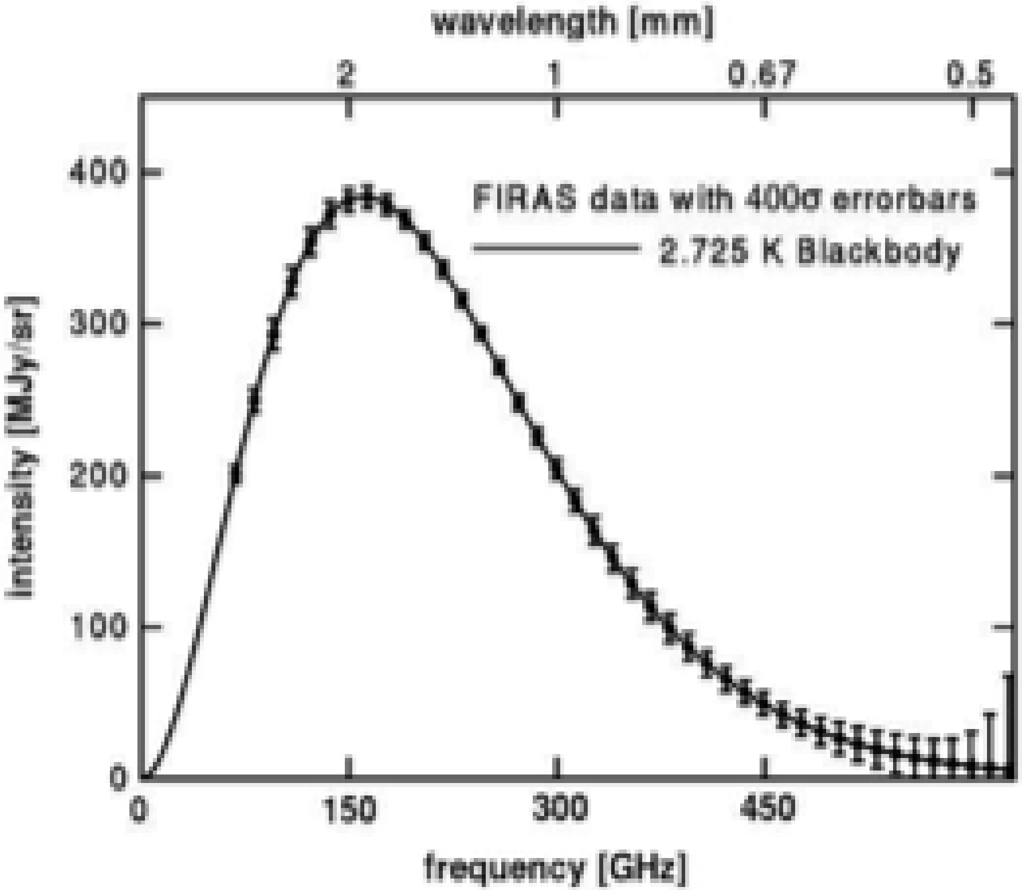


error uncertainties less than the width of the blackbody curve

T_{CMB} = 2.725 +/- 0.002 K

The perfect blackbody!

Far Infrared Absolute Spectrophotometer



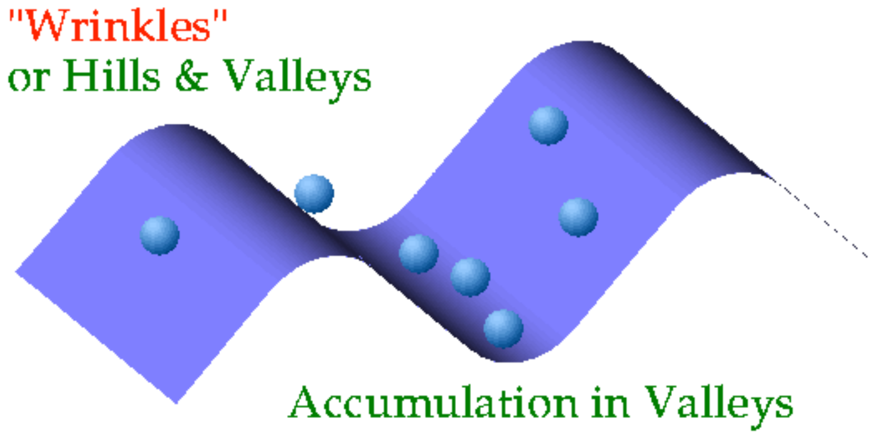
error uncertainties less than the width of the blackbody curve

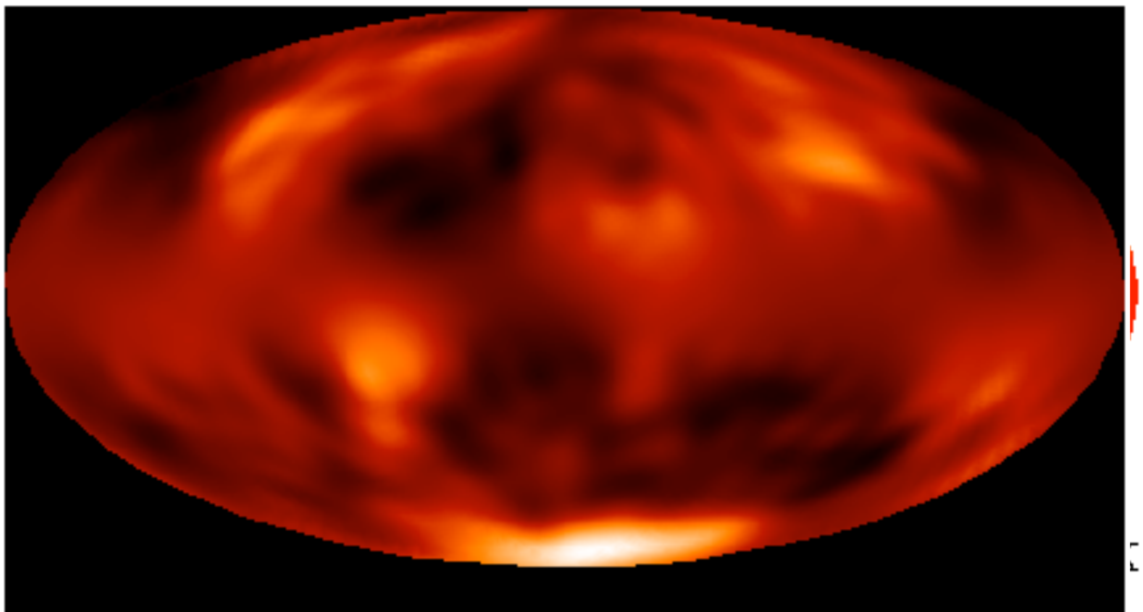
T_{CMB}= 2.725 +/- 0.002 K

Too difficult…

Where do the perturbations come from?

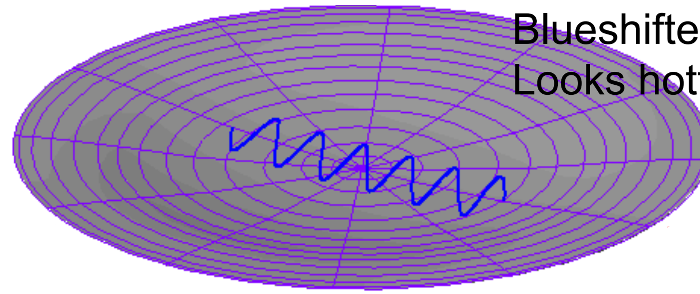
How is that we see the perturbations?



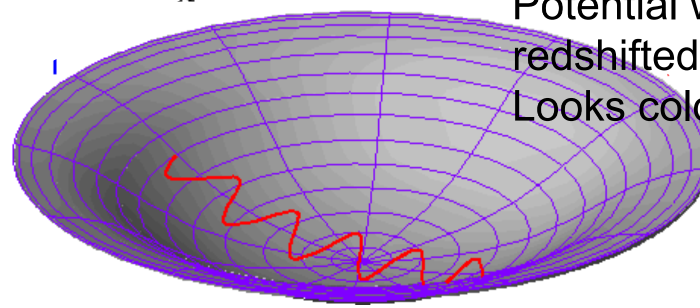


We see them like temperature

On scales larger than a degree, fluctuations were outside the Hubble horizon at decoupling



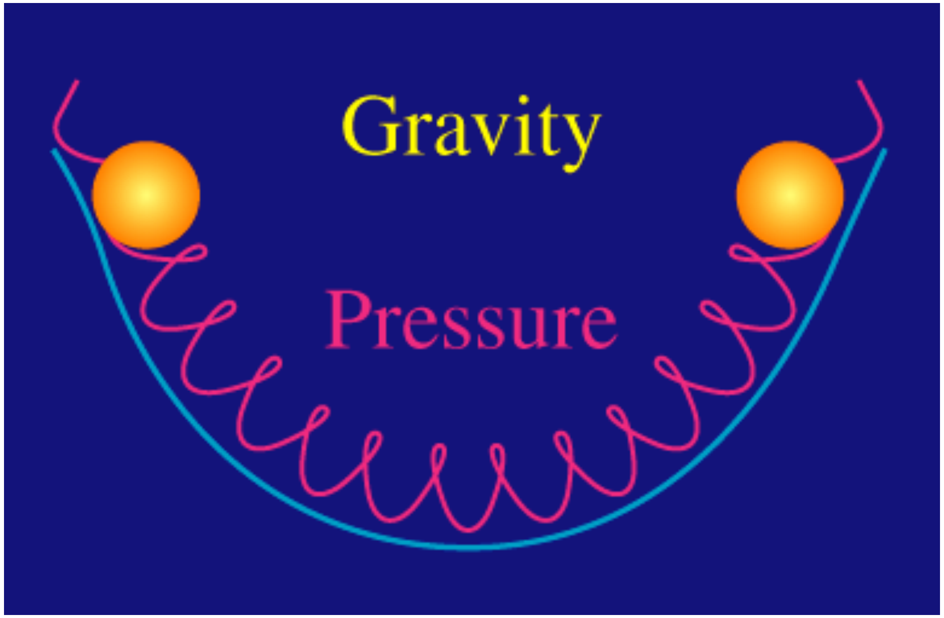
Photons need to climb out of potential wells before they can travel to us (redshift or blueshift) $h\nu/c^2\delta\phi$



$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\phi}{c^2} \longleftarrow \text{Given by } \delta\rho$$

We see them like temperature…

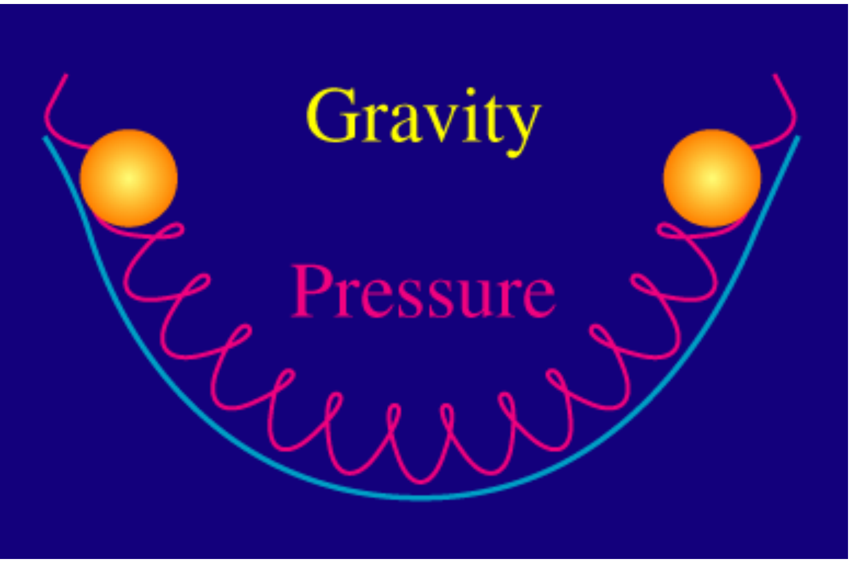
On smaller scales things get more complicated:
baryon and photons were coupled,
and photons oscillate (sound waves)

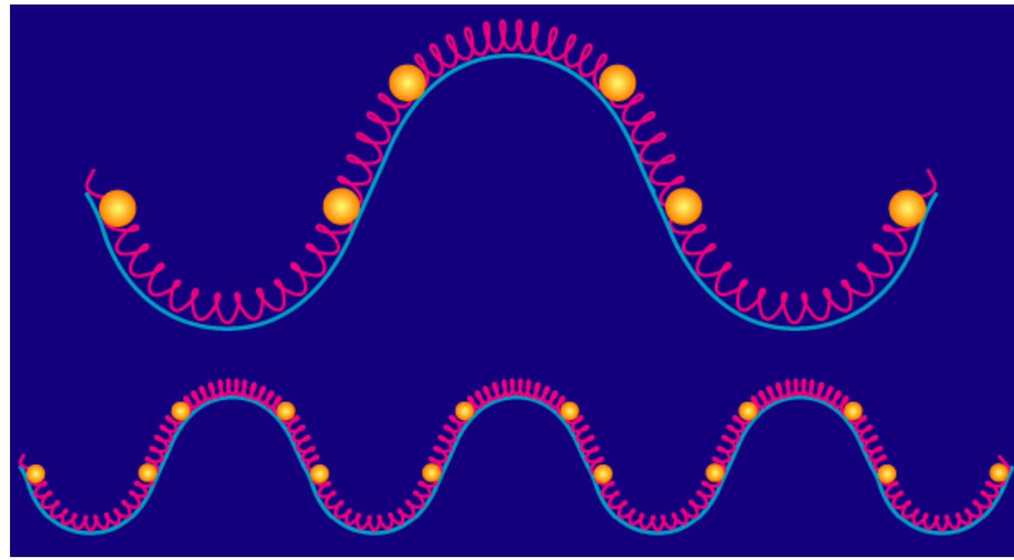


Horizon size at LSS --> Fundamental mode (over tones)

Until the universe is cool enough that H recombines and photons can travel freely to us, giving us a snapshot of the early Universe

There is time between when a perturbation enters the horizon (and starts oscillate)
And decoupling, when oscillations freeze…. And get imprinted in the CMB.





Longer wavelength modes oscillate slower

The frequency of the oscillation is equal to the wavenumber times the speed of sound: $\omega = kc_s$

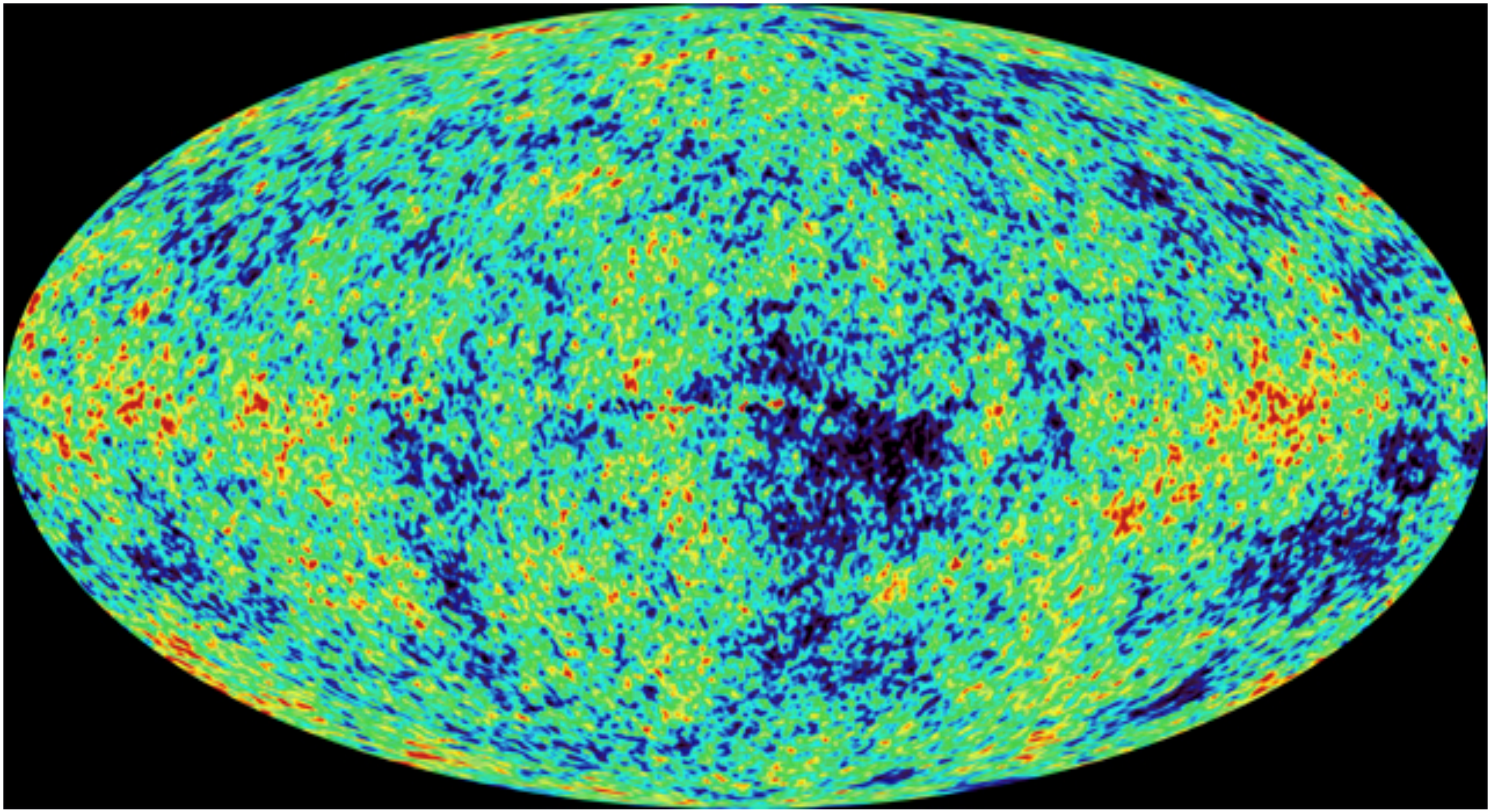
The largest scales (sound horizon) can only go through a compression,
Smaller scales can go through a compression and a rarefaction
Etc…

Animations courtesy of W. Hu

Can we see this in the sky?

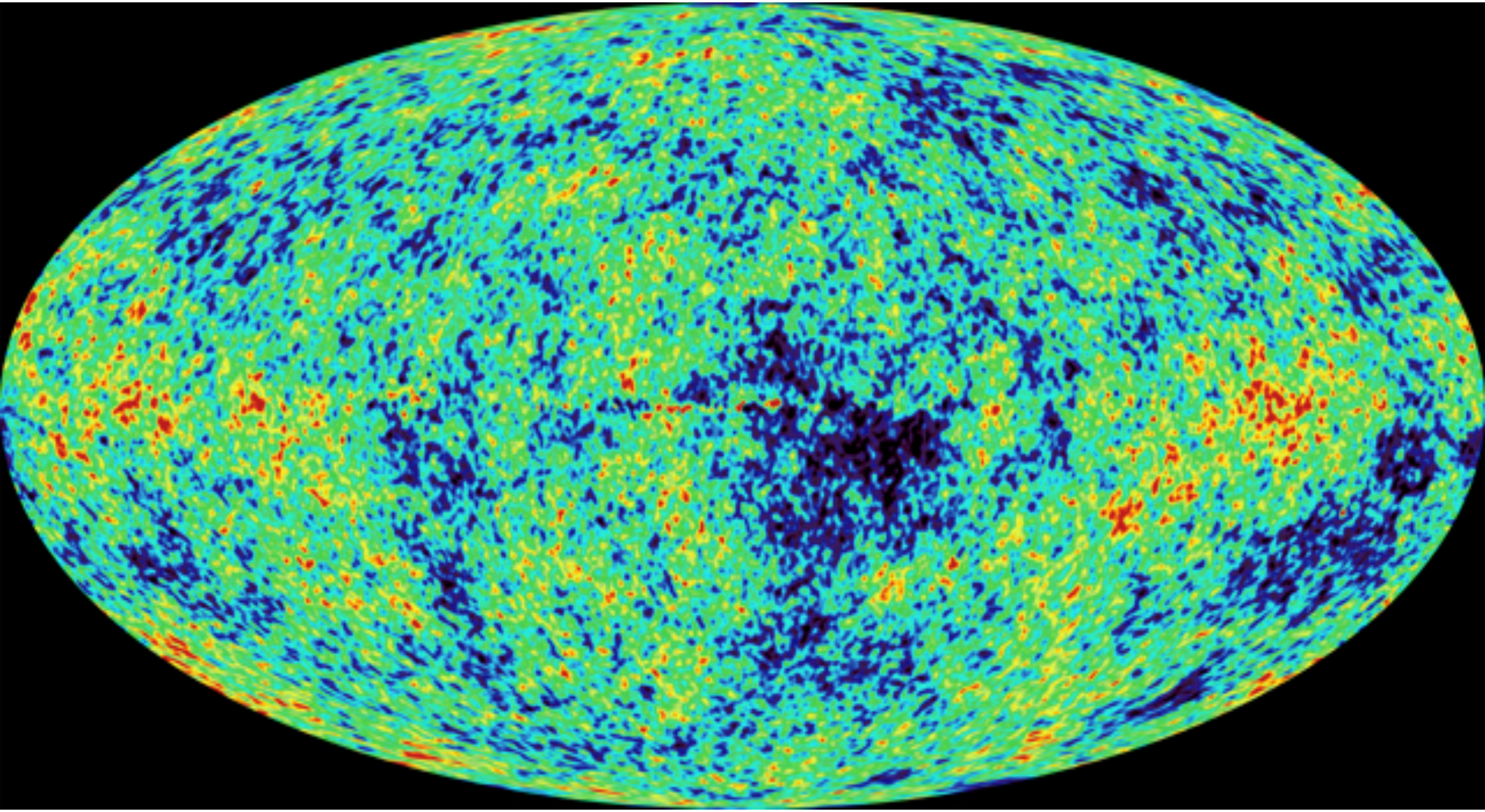
Seeing sound!

Cosmic symphony



Courtesy of WMAP

Hot and cold spots → Tiny ripples in density → seeds of galaxies



Detailed statistical properties of these ripples tell us a lot about the Universe

How’s that?

The Universe back then was made of a very hot and dense “gas”,
so it was emitting radiation

This is the radiation we see when we look at the CMB

Uniform , but with tiny (contrast x 100000) density (and temperature) ripples

Ripples in a gas? SOUND WAVES!

Truly a cosmic symphony… We are seeing sound!

These tiny fluctuations, quantitatively, give rise galaxies

We try to listen to the sound and figure out how the instrument is made

Fundamental scale→ Fundamental mode and overtones

like blowing on a pipe….

Power spectrum, again

$$\langle T \rangle = \frac{1}{4\pi}\int T(\theta,\phi)\sin\theta d\theta d\phi = 2.726K$$

Challenge!

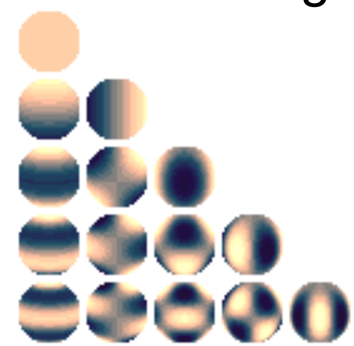
$$\frac{\Delta T}{\langle T \rangle} = \frac{\Delta T}{T}(\theta,\phi) = \frac{T - \langle T \rangle}{\langle T \rangle}$$

rms 1 in 10^5

Even bigger challenge!

Expand in spherical harmonics

$$\frac{\Delta T}{\langle T \rangle} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm} Y_l^m(\theta,\phi)$$

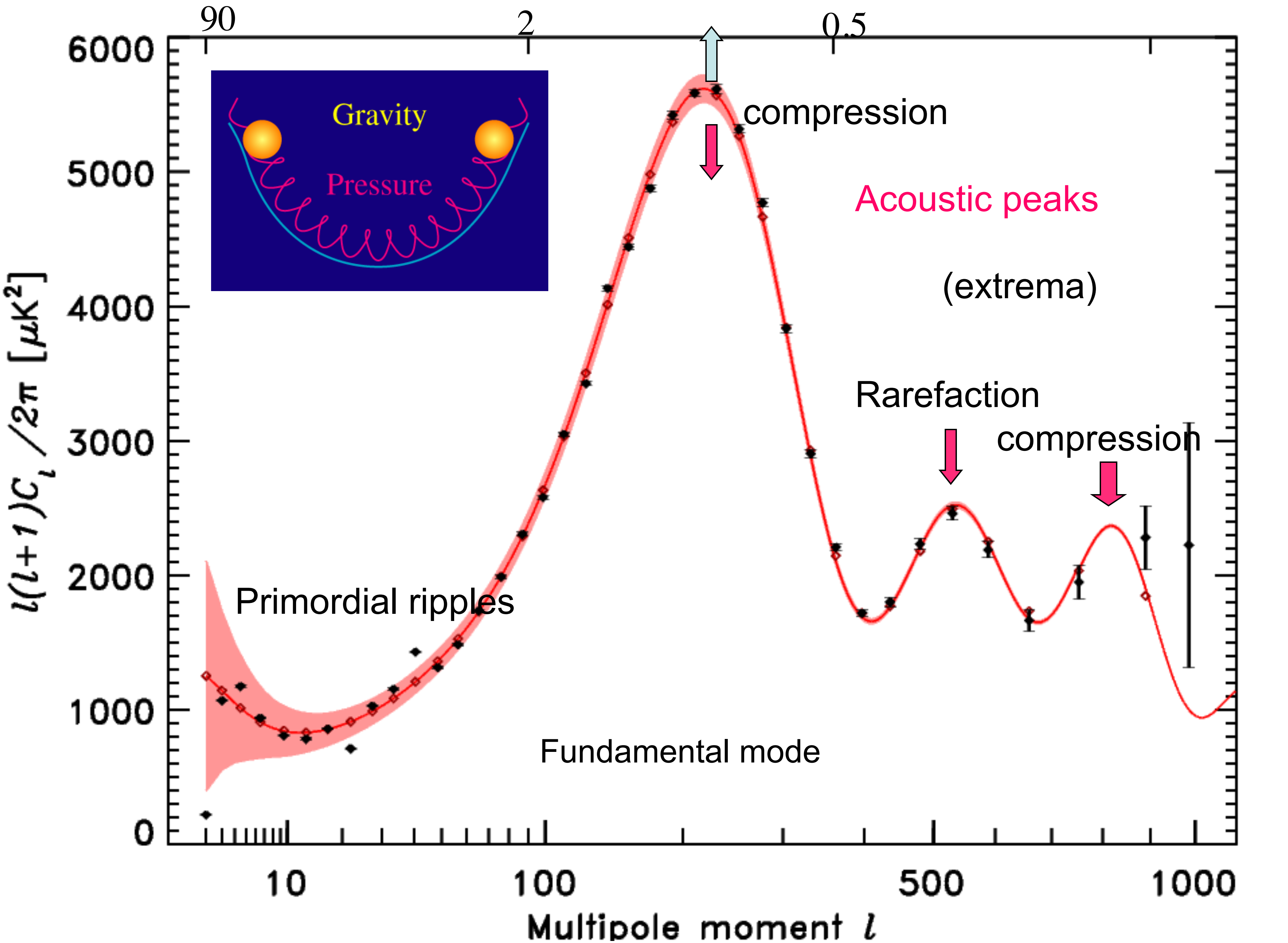


$$C(\theta) = \langle \frac{\Delta T}{T}(n)\frac{\Delta T}{T}(n')\rangle = \frac{1}{4\pi}\sum_l (2l+1) C_l P_l(\cos\theta)$$

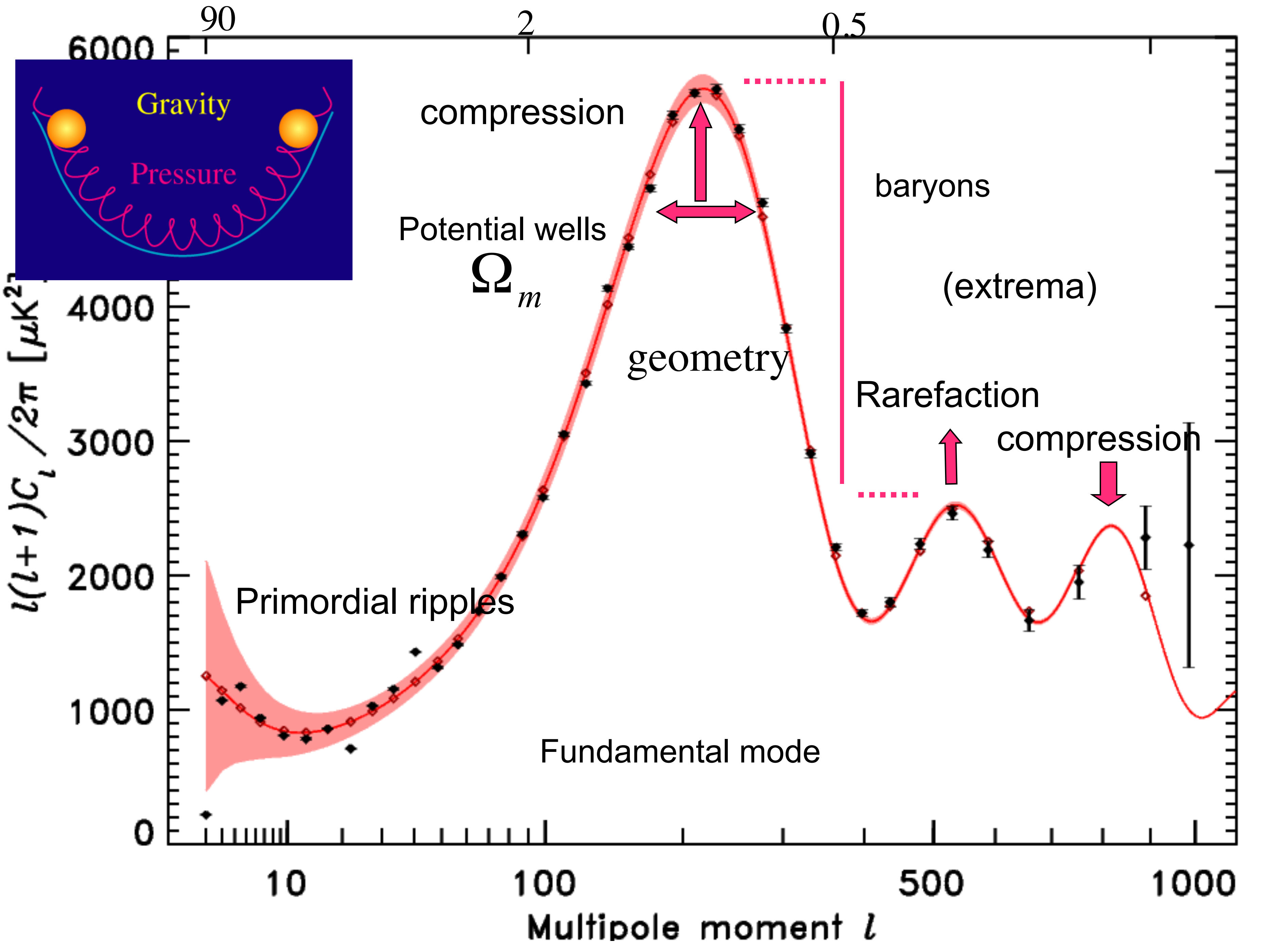
Legendre polynomials

$$C_l = \frac{1}{2l+1}\sum_l |a_{lm}|^2$$

Angular power spectrum

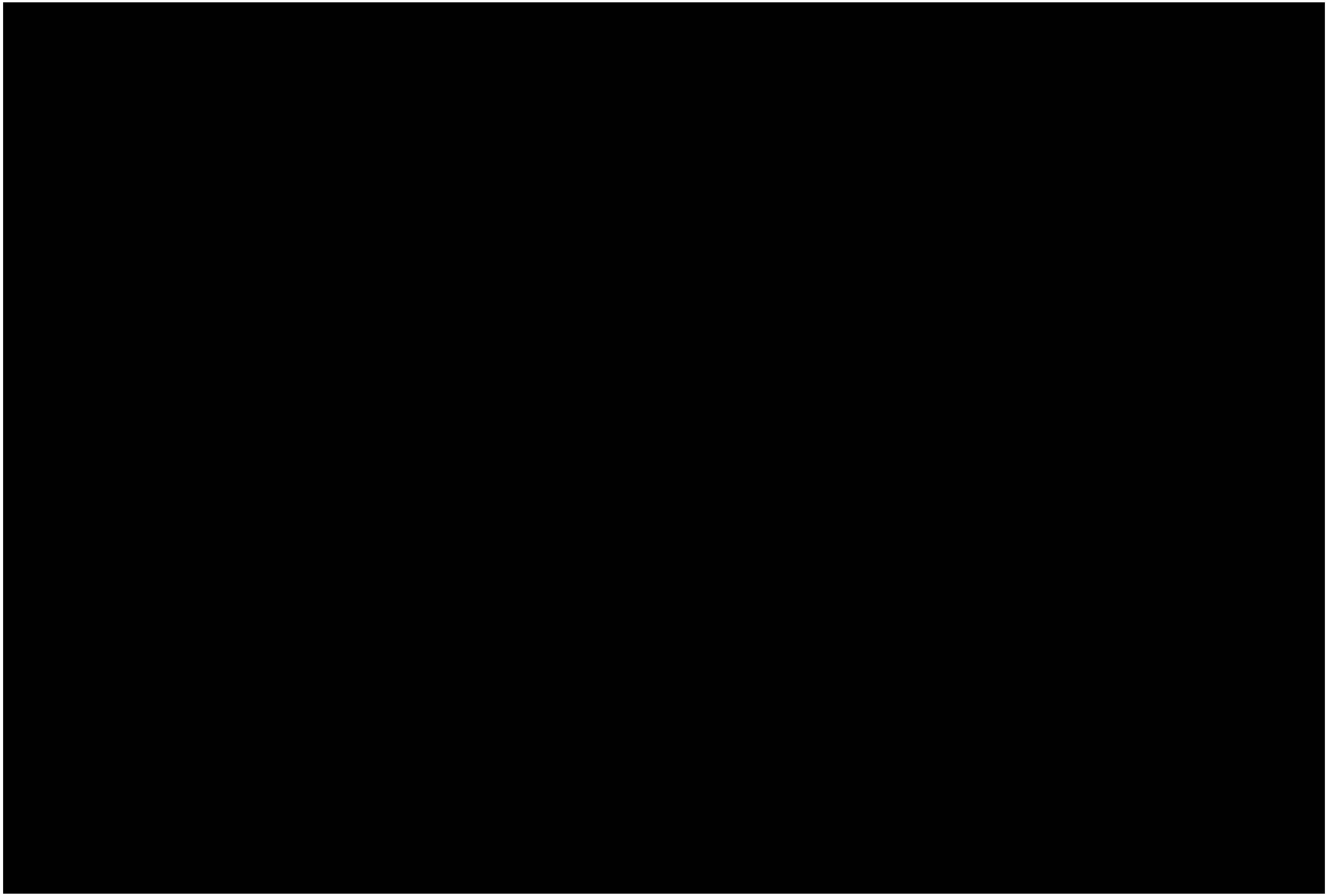


90
2
0.5
6000
5000
4000
3000
2000
1000
0
Gravity
Pressure
compression
Acoustic peaks
(extrema)
Rarefaction
compression
Primordial ripples
Fundamental mode
$l(l+1)C_l/2\pi\ [\mu K^2]$
10
100
500
1000
Multipole moment l



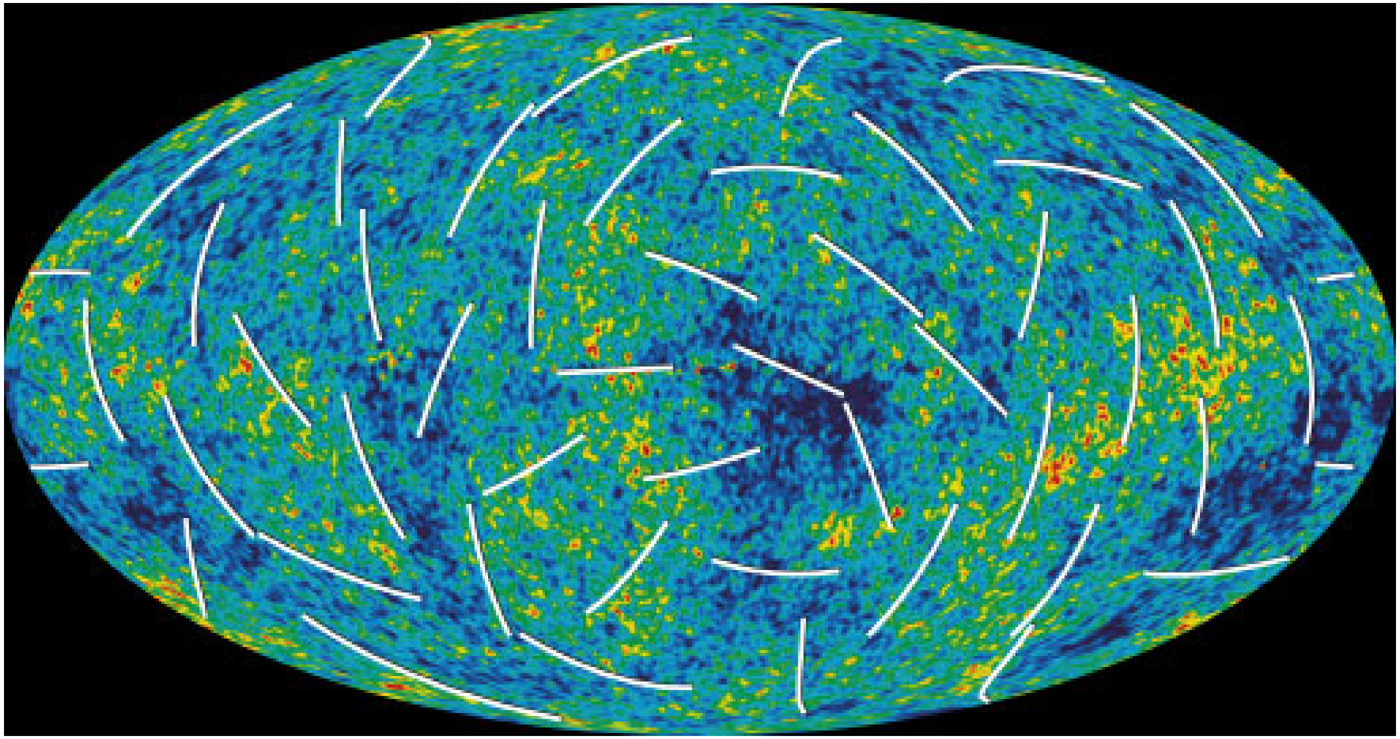
90
2
0.5
6000
5000
4000
3000
2000
1000
0
Gravity
Pressure
compression
Potential wells
Ω_m
geometry
baryons
(extrema)
Rarefaction
compression
Primordial ripples
Fundamental mode
$l(l+1)C_l/2\pi\ [\mu K^2]$
10
100
500
1000
Multipole moment l


Courtesy of WMAP

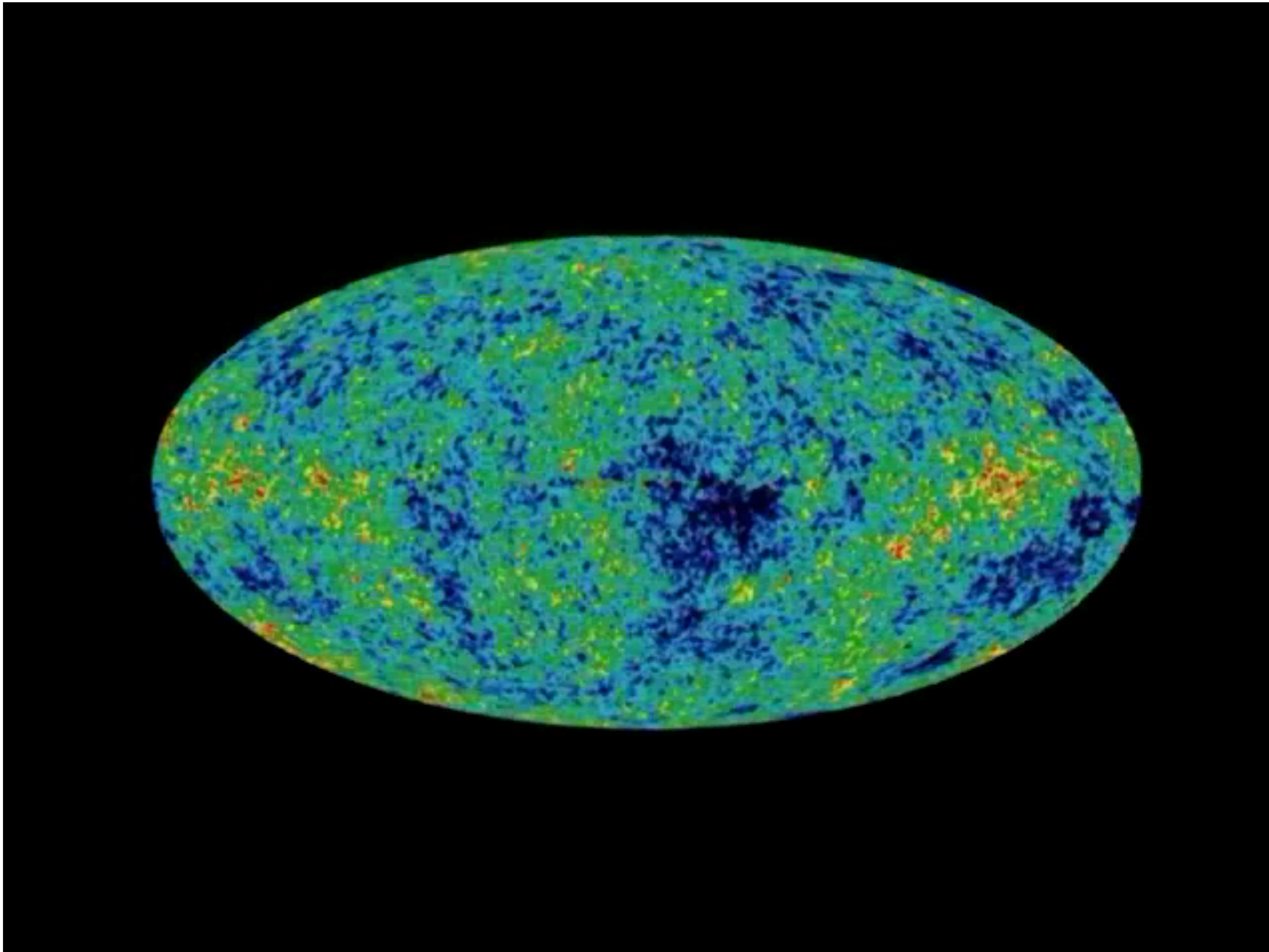


Courtesy of WMAP

Just because you asked:

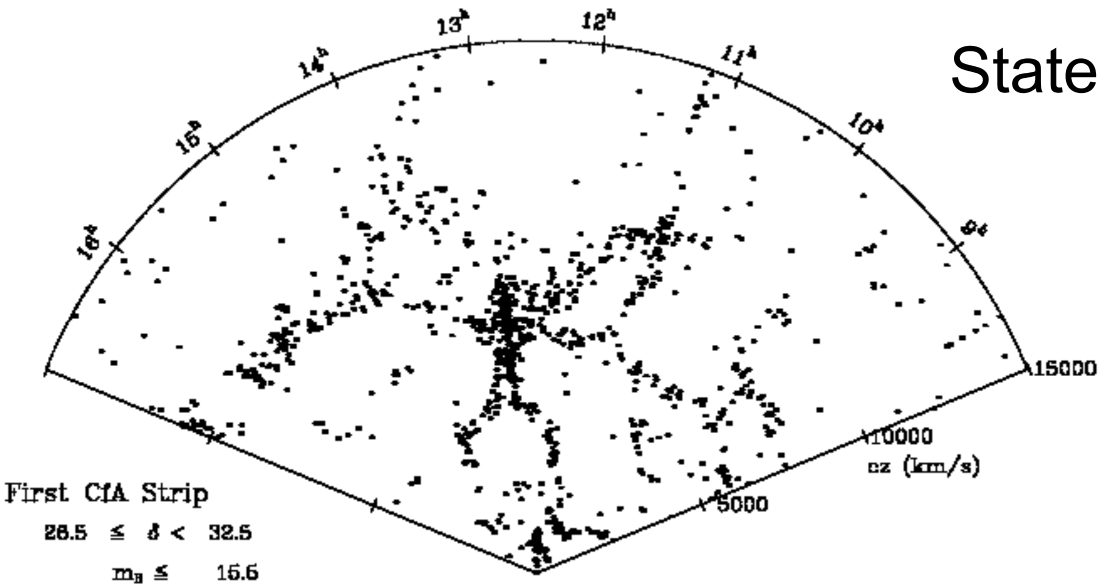


There is extra information:polarization



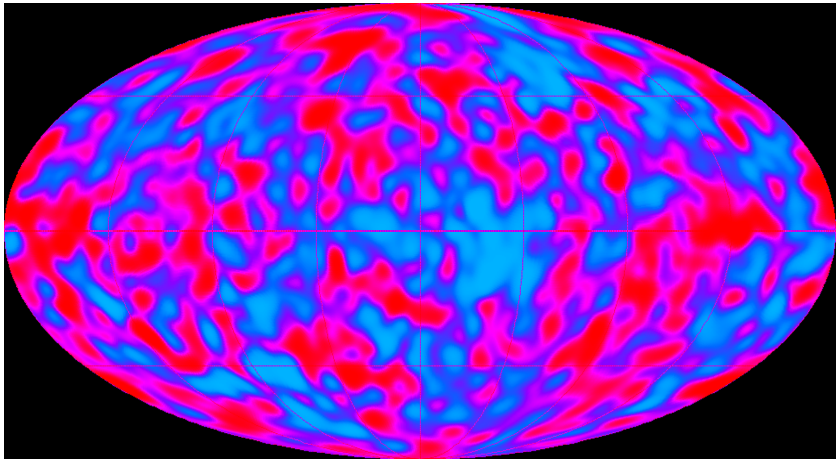
Courtesy of WMAP

Extremely successful standard model for cosmology



State of the art of data then…

~14 Gyr
(a posteriori information)

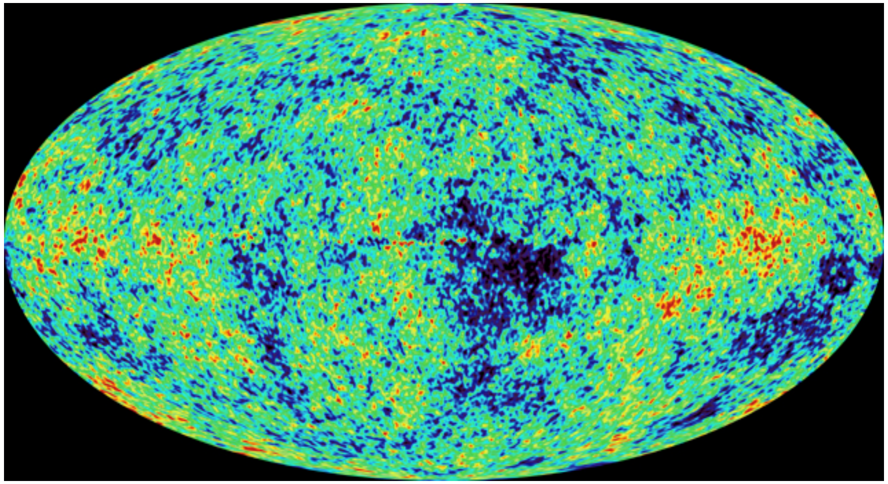


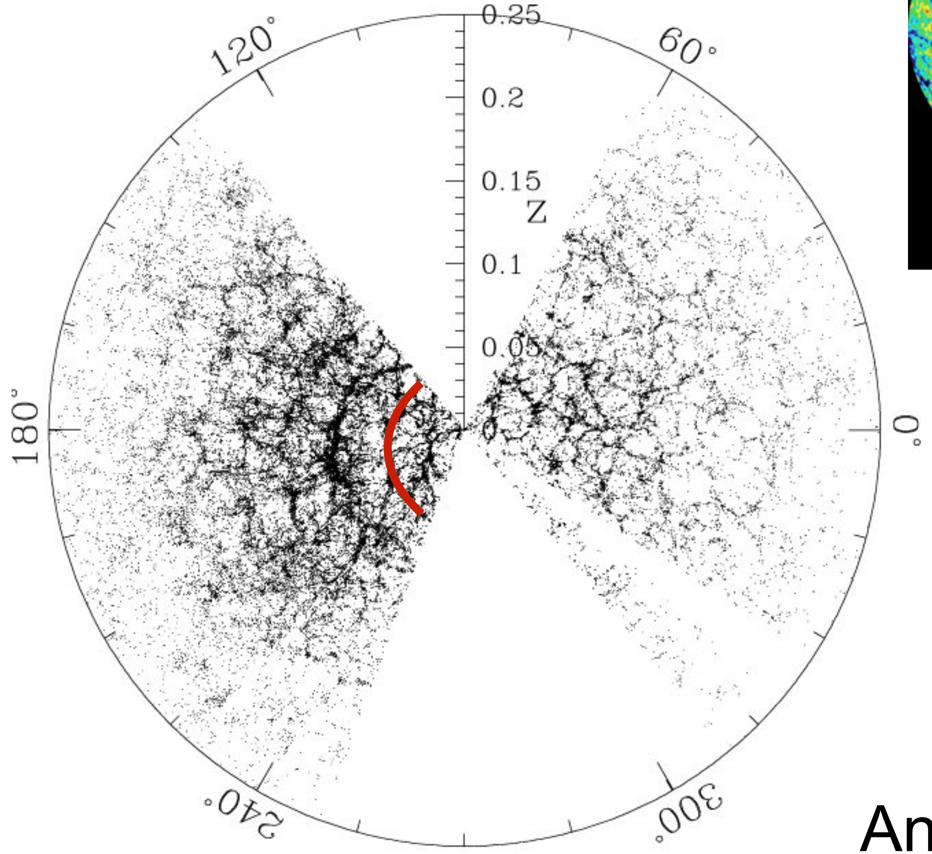
(DMR)COBE

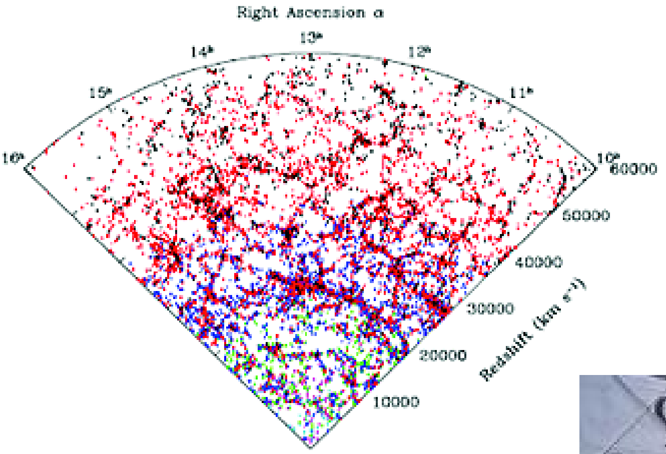
CMB

380000 yr
(a posteriori information)

Avalanche of data







And it still holds!

The era of precision cosmology:

LCDM: the “standard” model for cosmology

Few parameters describe the Universe composition and evolution

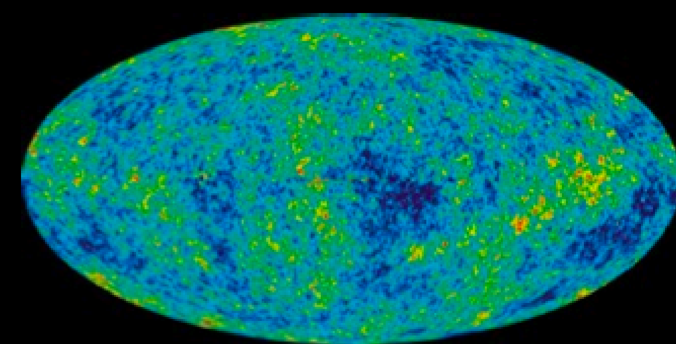
Homogenous background

$\Omega_b, \Omega_c, \Omega_\Lambda, H_0, \tau$

- atoms 4%
- cold dark matter 23%
- dark energy 73%

Λ? CDM?

Perturbations

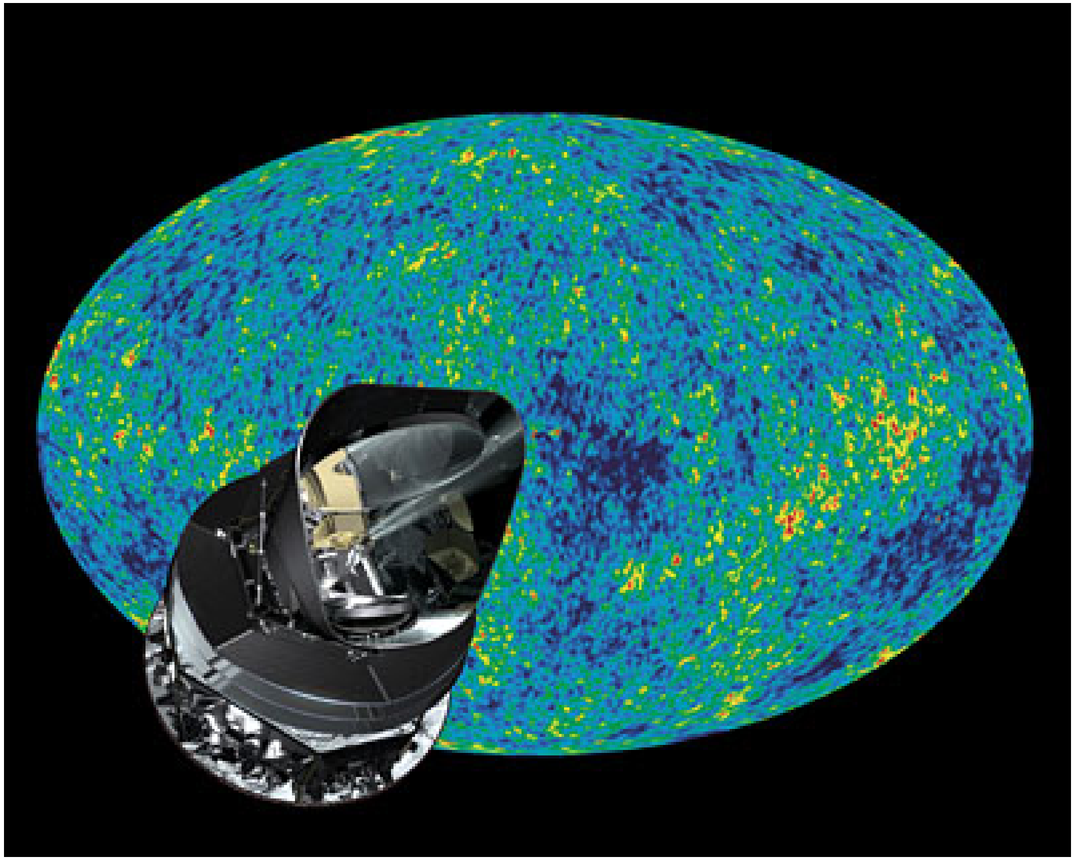


A_s, n_s, r

- nearly scale-invariant
- adiabatic
- Gaussian

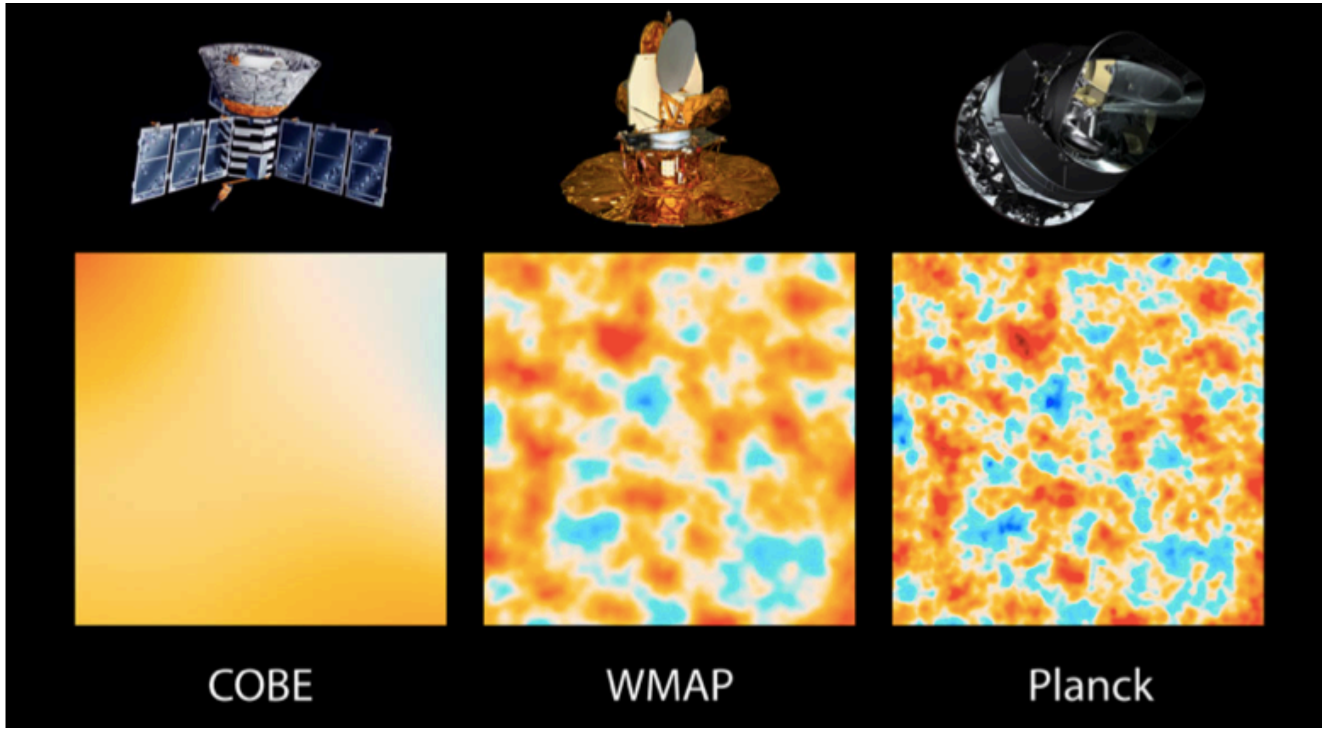
ORIGIN??

Planck (results announced this year!)

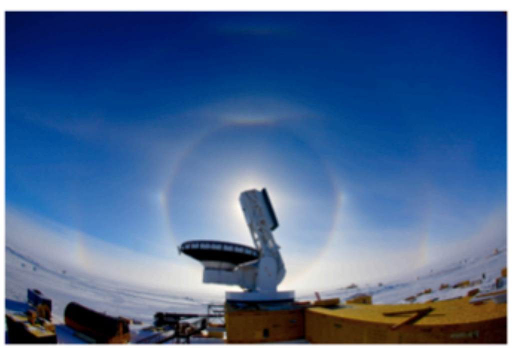


The ultimate experiment for primary CMB temperature

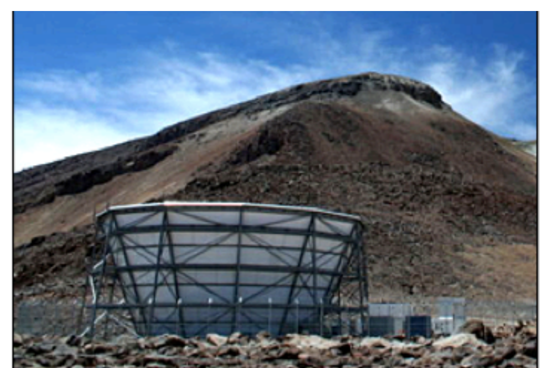
Satellites: full sky



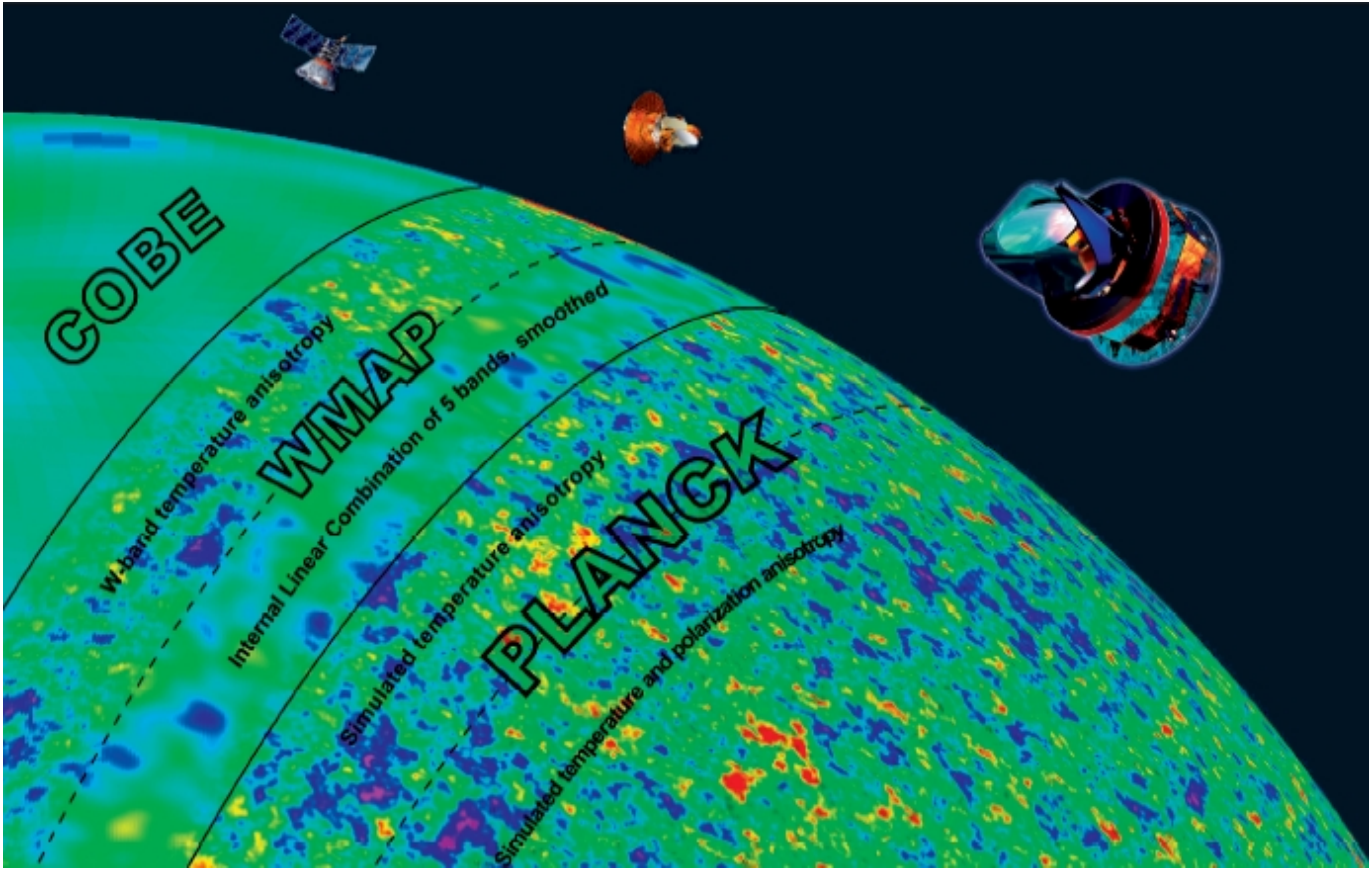
Ground-based experiments (not full sky but better resolution)



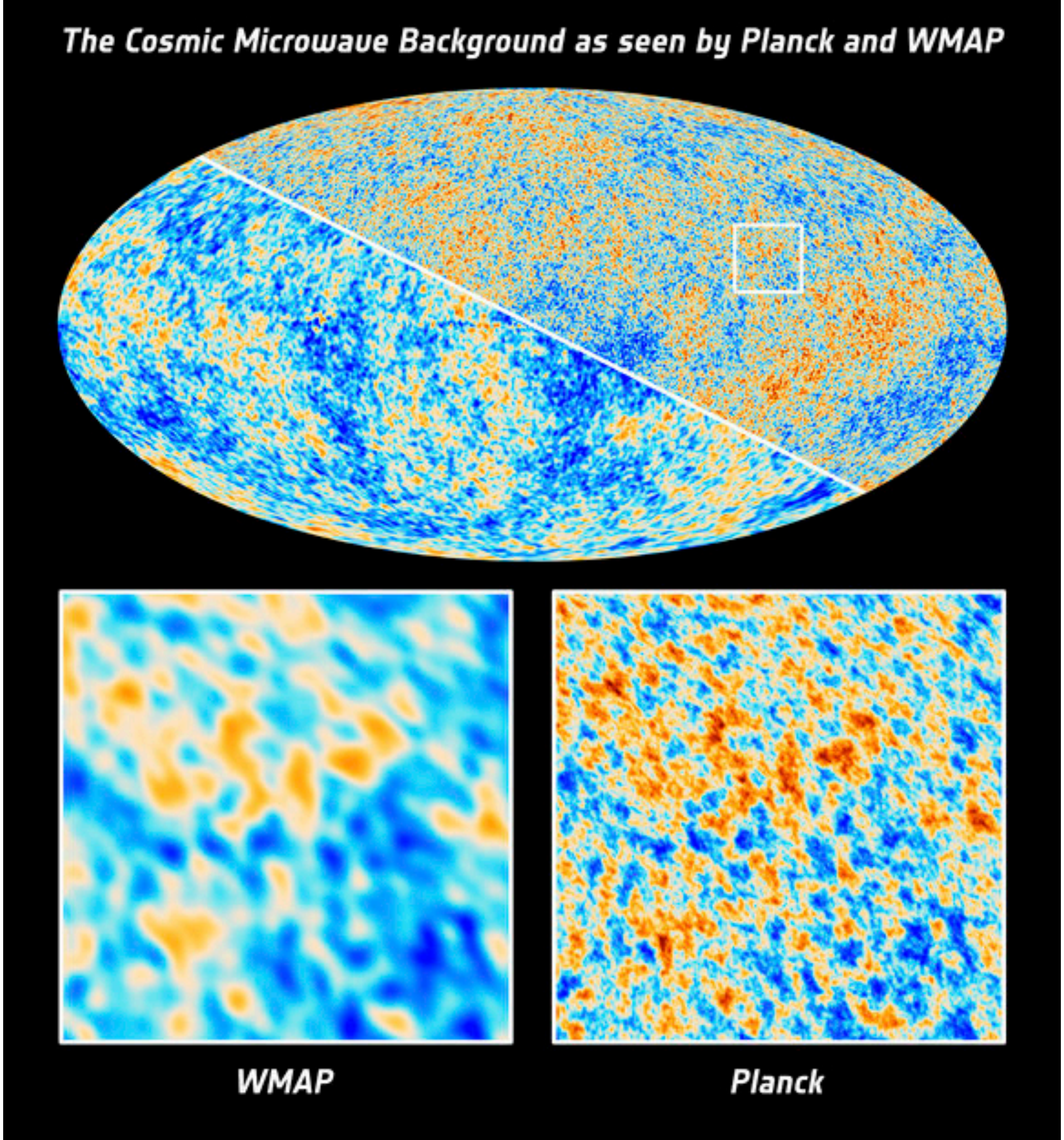
South Pole Telescope (SPT)



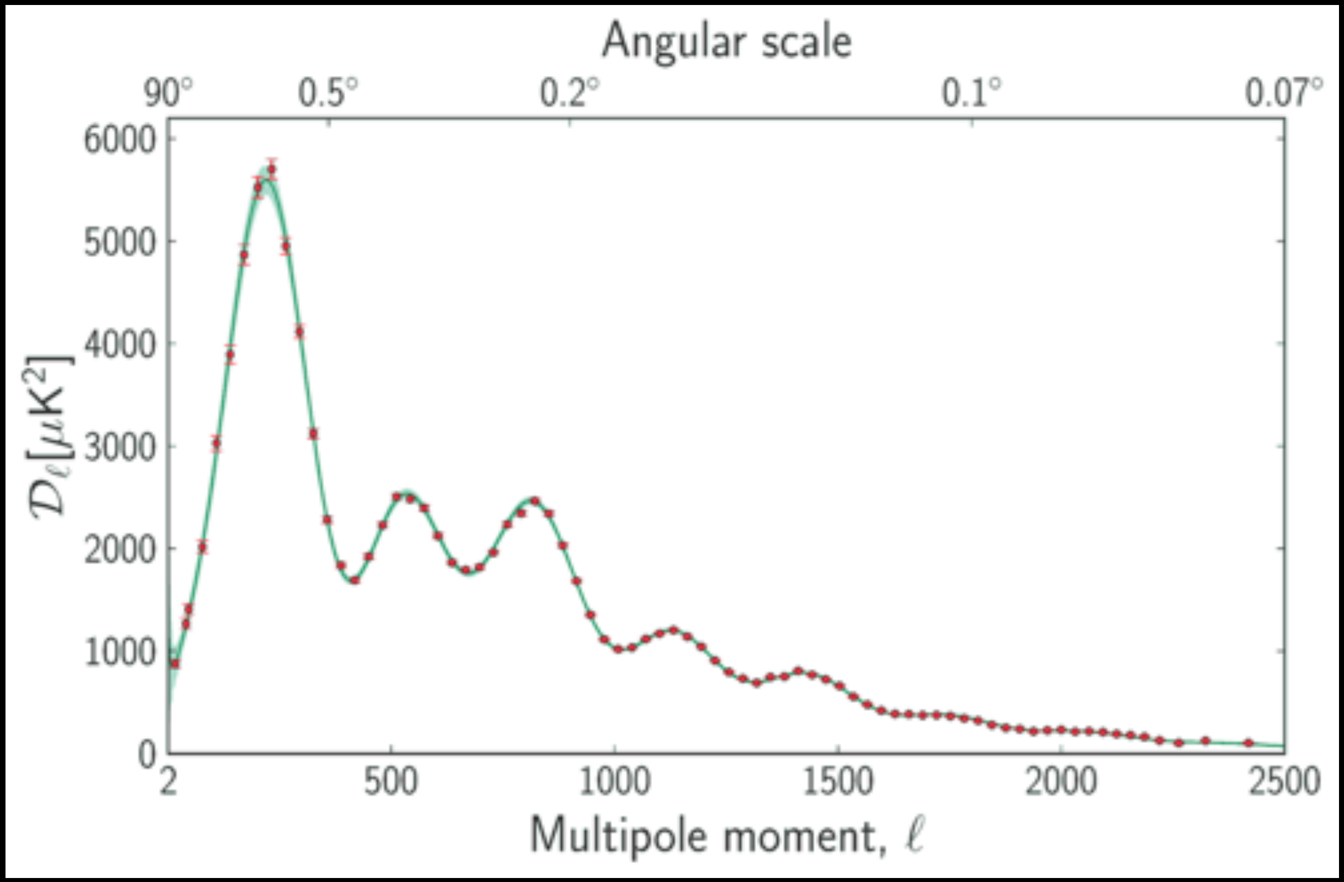
Atacama Cosmology Telescope (ACT)



COBE
W-band temperature anisotropy
WMAP
Internal Linear Combination of 5 bands, smoothed
Simulated temperature anisotropy
PLANCK
Simulated temperature and polarization anisotropy

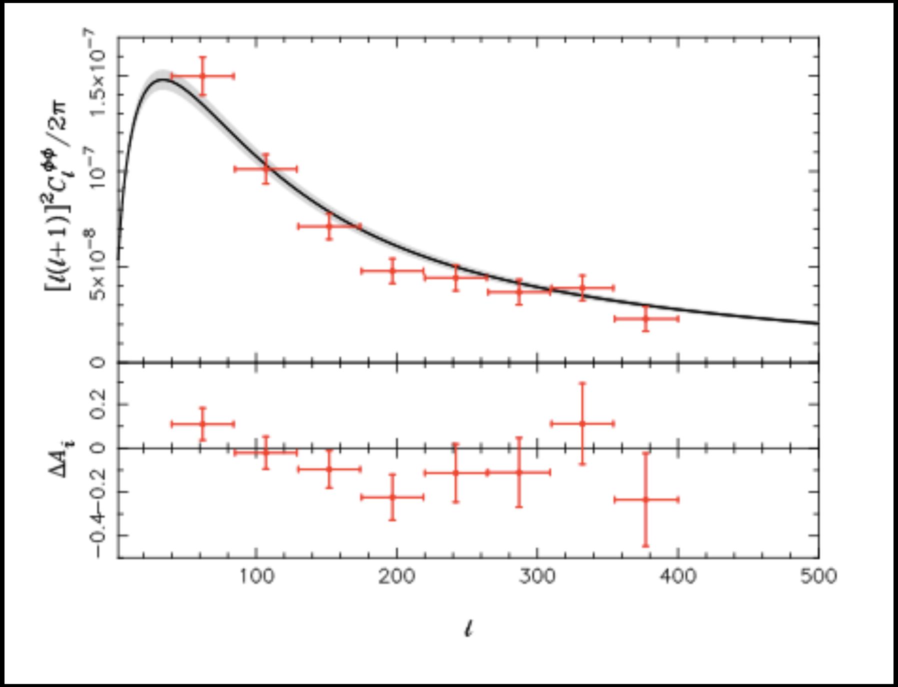

The Cosmic Microwave Background as seen by Planck and WMAP
WMAP
Planck

Planck

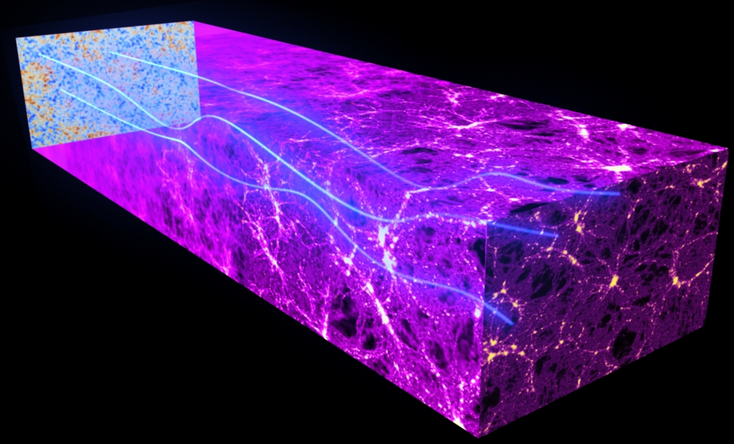


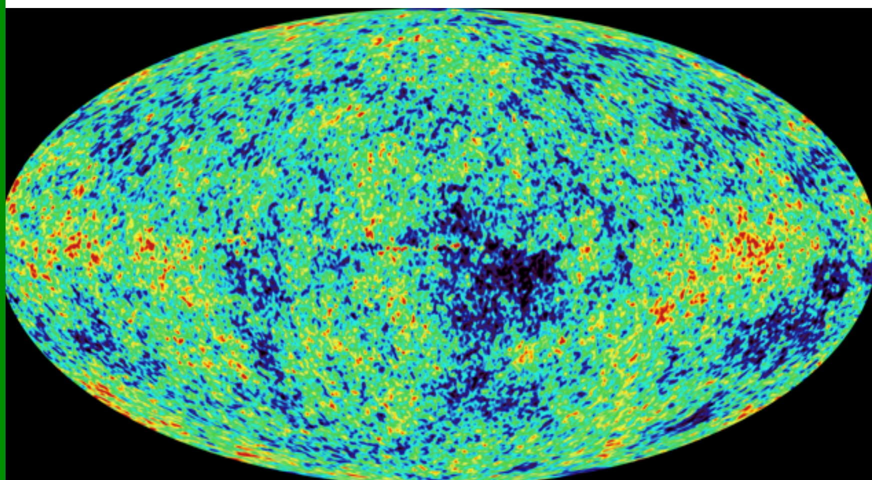
Planck collaboration , 2013, paper XVI

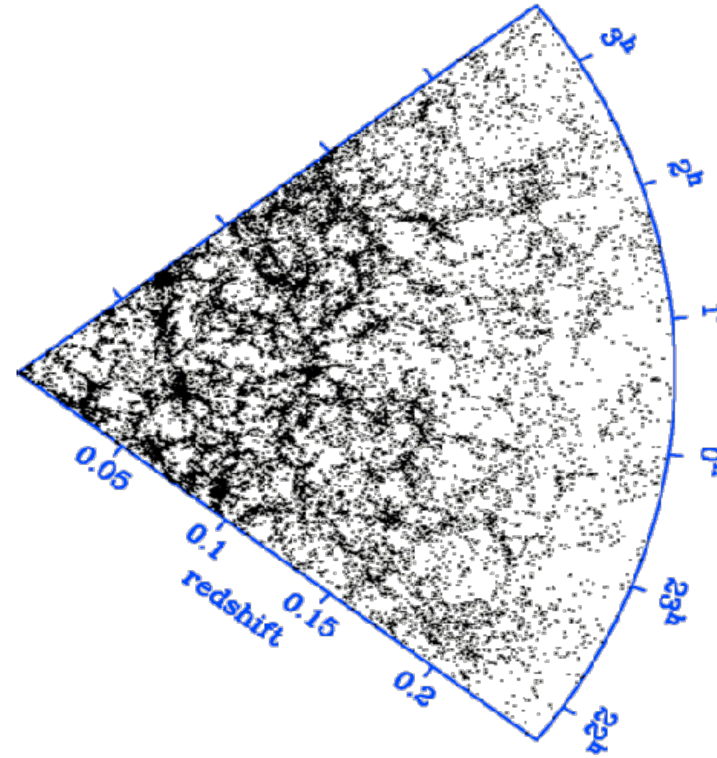
NEW measurement

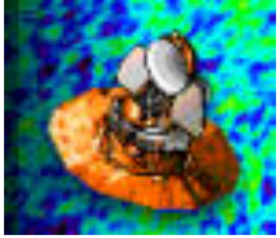


Planck collaboration , 2013, paper XVI



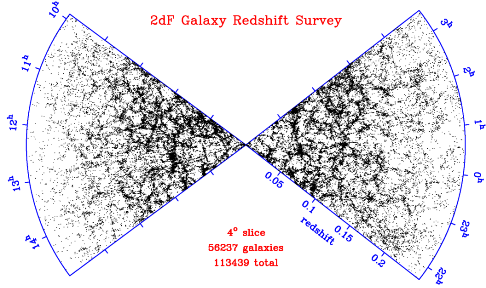


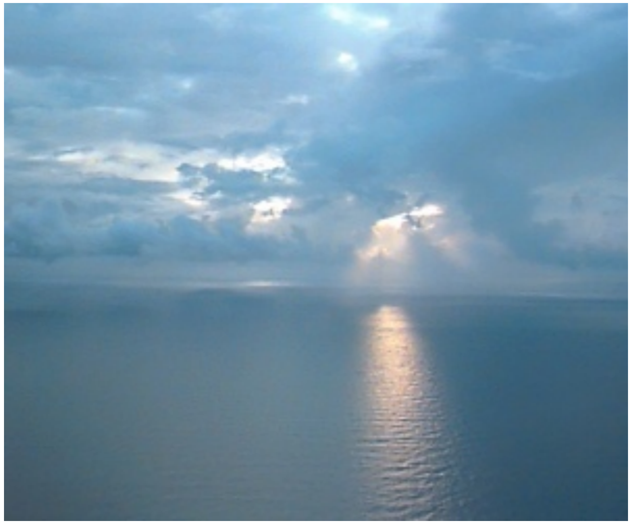

3h
2h
1h
0h
23h
22h
0.05
0.1
0.15
0.2
redshift

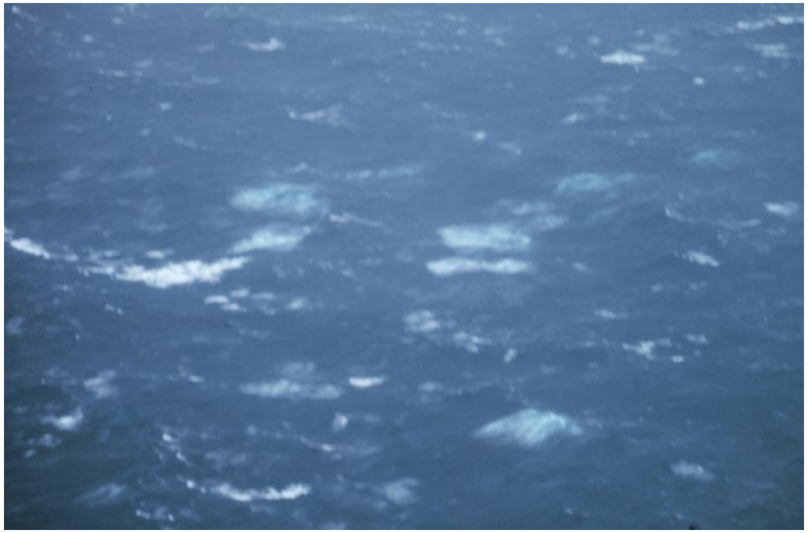


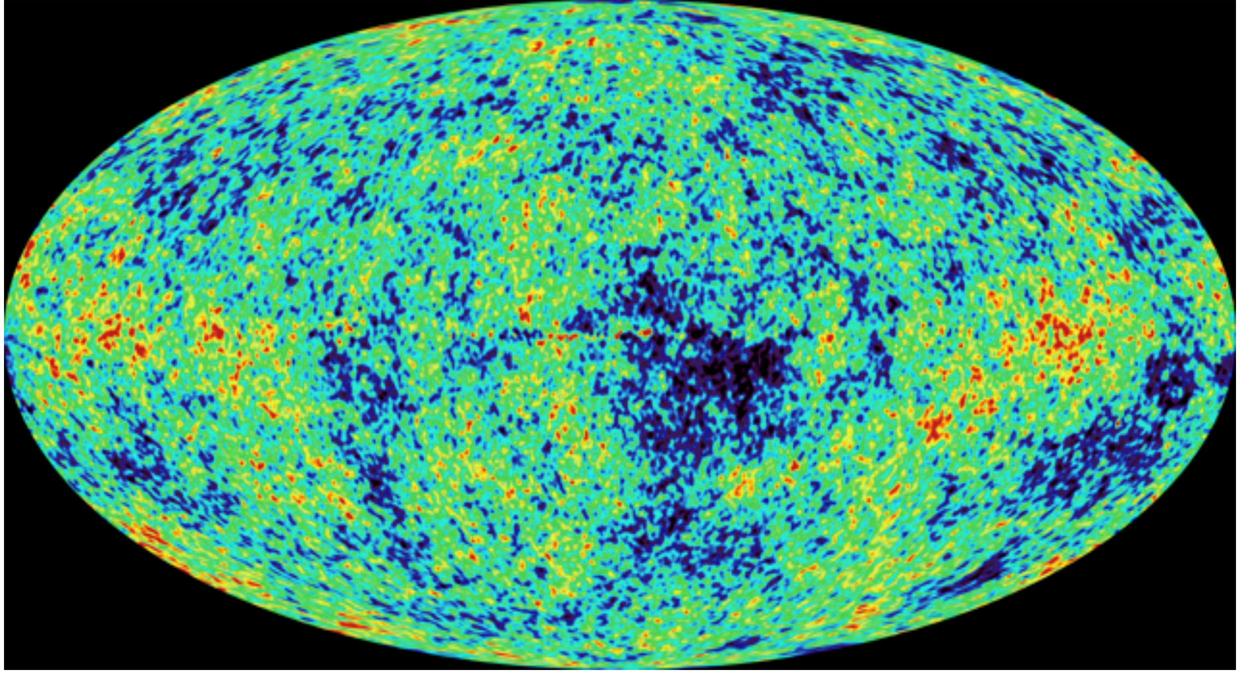
HOW?

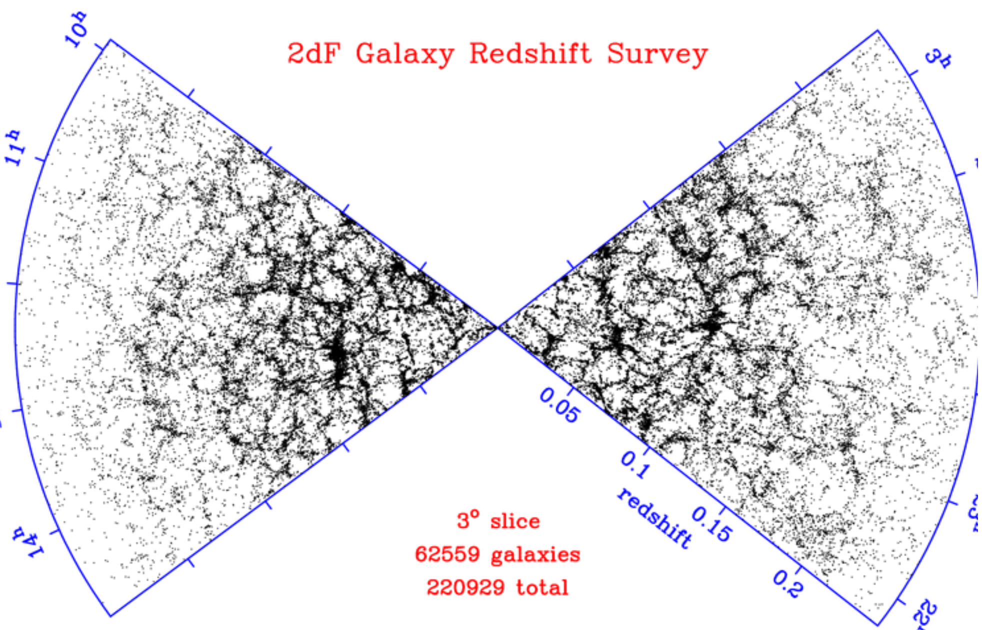
Tools:statistics.

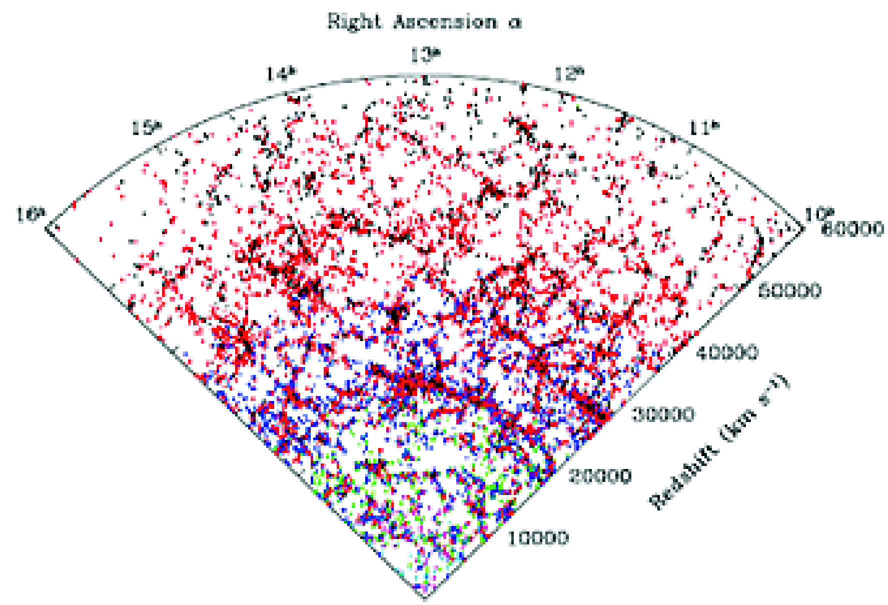




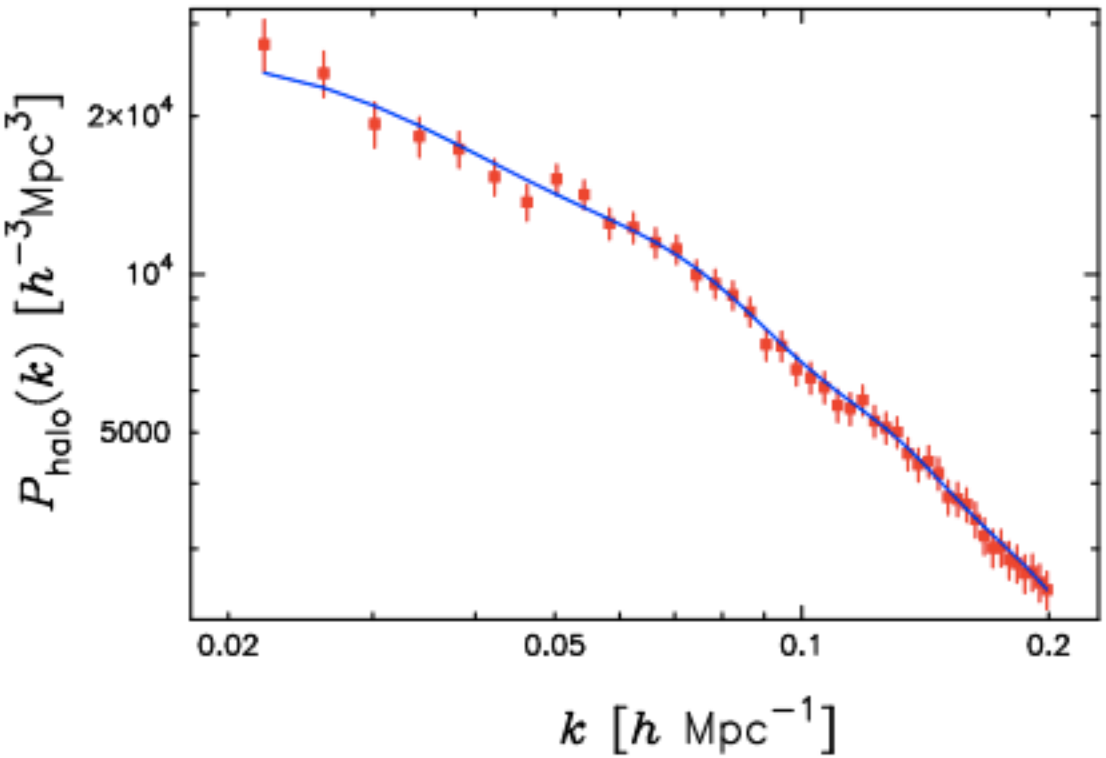





2dF Galaxy Redshift Survey
10^h
11^h
12^h
13^h
14^h
3^h
22^h
0.05
0.1
0.15
0.2
redshift
3° slice
62559 galaxies
220929 total


Right Ascension α
14^h
13^h
12^h
15^h
11^h
16^h
10^h
60000
50000
40000
30000
20000
10000
Redshift (km s^{-1})

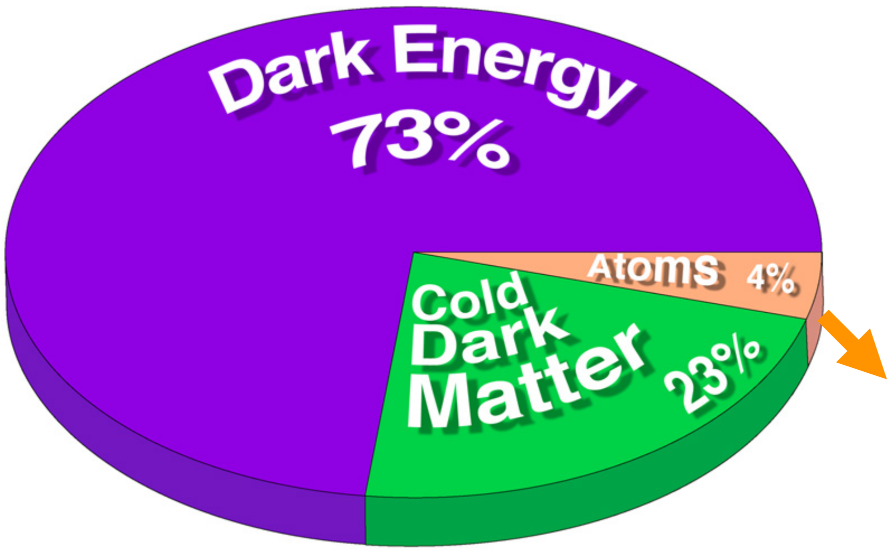
This is not a fit to the data



Data: Reid et al 2010:Large-scale structure power spectrum probed by a galaxy survey. SDSS (DR7)

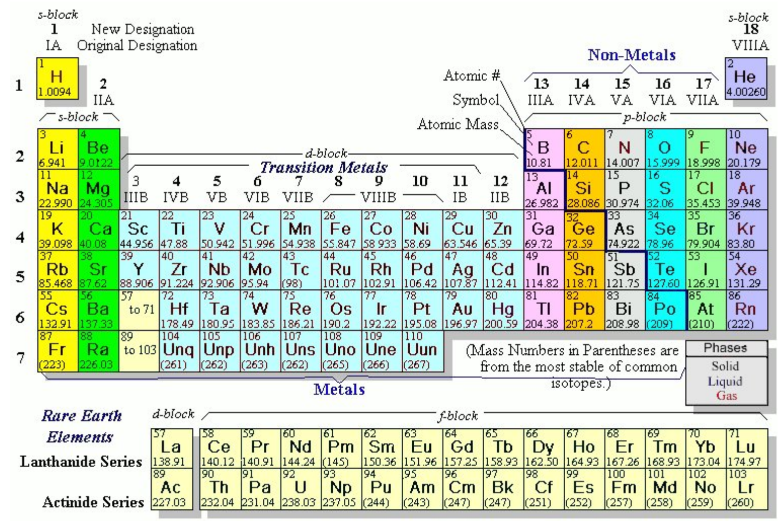
Line: standard cosmological model prediction
 from CMB Planck observations!

The standard cosmological model



ΛCDM (or LCDM)

+flatness+scale invariance



Cosmology has now a standard model:

6 parameters fit observations of the Universe from z=1100 to z=0

- Basic parameters are accurately determined
  - Many can be measured using multiple techniques
  - CMB best fit now consistent with other measurements

The era of precision cosmology

A joke until 15 years ago or so…

Cosmological parameters measured with % precision

Have model that describes well the observations of the universe from 380,000 years after the big bang to today, 13.7 Billion years (10^9) after the big bang. And even describe what happens right through factions of seconds after the big bang.

Prediction of CMB
prediction of its fluctuations
Agreement between theory and observations!

The standard model of cosmology!

Still…

Last Judgment, Vasari, Florence Duomo

Key concepts

•The content of the universe (amount and type of stuff) govern its geometry and expansion history

•Friedmann Equations (with these you go everywhere!)

•The CMB and large-scale cosmological structures